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Research Note 1 November 1, 1981

Position of the Increment Borer for Accurate Basal Area Growth Determination

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ABSTRACT

Tree cross-sections and increment cores were examined to determine the position for the borer that results in the most accurate basal area growth figures. The analysis applies in situations where either caliper or D-tape is used to measure final stem size. Results are included for trees with eccentric boles and trees with circular boles. The recommended cores to be used for basal area growth determination are: a core taken under a point of average crown length (when crown length is asymmetric) on trees with circular boles; and the average of two cores taken at either end of the minor axis for trees with eccentric stems.

Introduction

Increasing recognition in California of the need for basal area and volume growth data from young-growth stands has led to an increased emphasis on the establishment of permanent plots. In stands of interest to the forest manager where growth information is not available from existing plots, trees are often bored for periodic growth. Cores are generally taken at breast height, and a number of rules exist for the position of the boring around the bole (Husch 1963, Meyer 1953). This study focuses on the accuracy of growth predictions computed from increment cores. The study is divided into two sections: trees with eccentric boles, and trees with circular boles.

Most of the work done in the area of increment core procedure has been concerned with minimizing the variance of radial growth estimates computed from increment cores. Matern (1951) and Siostrzonski (1958), testing from one to four cores, found that two cores taken from opposite sides of the tree provided the most efficient estimate. Amidon and Dolph (1980) investigated the advantage of two cores, instead of one, when the increment is to be used in a least squares estimation. But there does not seem to be any information on the position of the core around the bole of the tree for the most accurate estimation of basal area growth.

Part One: Trees with Eccentric Boles

Physiology

Gray (1956) discussed the structural advantages of the eccentricity of a tree stem when lateral forces are primarily in one direction. These forces may be related to the slope of the land, an asymmetric crown, persistant uni-directional winds, or the close proximity of a neighboring tree. The eccentricity results from the formation of reaction wood, which is initiated when these forces cause the tree to lean in one direction more than any other.

The plant growth substance, auxin, which tends to move with gravity, controls the formation of reaction wood. In gymnosperms an increase in auxin in the lower side of the stem causes the formation of compression wood. In angiosperms it seems the formation of tension wood on the upper side of the stem is associated with a decrease in auxin in the upper side of the stem (Bidwell 1974).

Geometric Model for the Eccentric Tree

Caliper

Calipers have been recommended as the instrument of choice when measuring an out-of-round tree (Assman, 1970). Assuming an eliptical shape the basal area of the tree is given by

$$A = \frac{\pi}{144}\left(\frac{a}{2}\right)\left(\frac{b}{2}\right) = \frac{\pi}{144}\left(\frac{ab}{4}\right)$$

Where a and b are the major and minor axes [1/] (the largest and smallest perpendicular diameters--in inches), and A is the cross sectional area (ft^2).

Thus the basal area growth (BAG) is given by the difference between two elipses

$$BAG = A_2 - A_1$$

or

$$BAG = \frac{\pi}{4 \times 144}(a_2b_2 - a_1b_1)$$

where a_1, b_1, a_2, b_2 represent the major and minor axes of the tree at two points in time.

Since the use of calipers will give exact basal area figures when the tree is a true elipse and a geometric mean diameter is computed, an elipse is often used as the model for an eccentric tree. Three possible representations of basal area growth when the tree stem is eliptical are shown in figures 1a,b,c.

[1/] The major and minor axes used here are the calipered diameters of the section. In many mathematics texts the axes are defined to be one-half of these diameters.

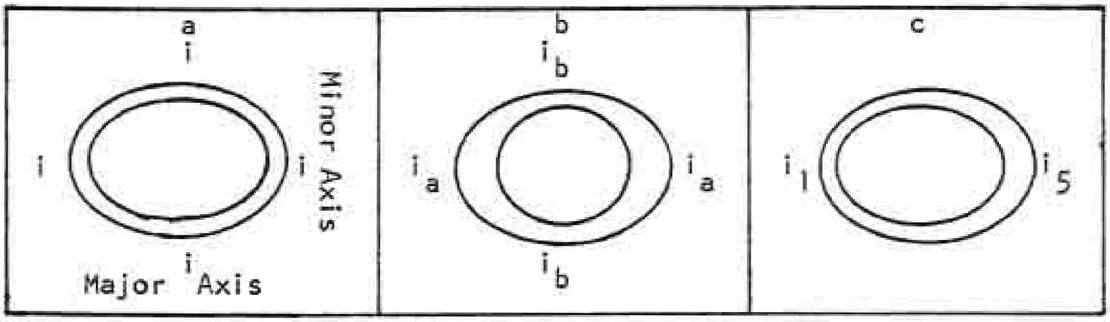

Figure 1. Representations of growth for elliptical stems

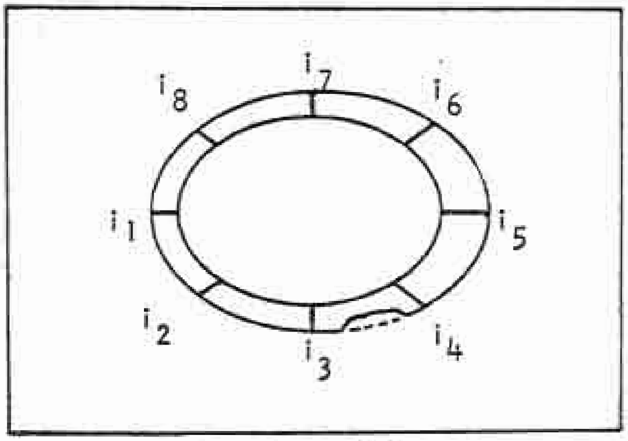

Figure 2. Locations of increment cores on test trees

In figure 1a, it is assumed that increment does not vary around the stem. Basal area growth can be computed by means of equation 1a from a single core taken along either the major or minor axis.[1/]

$$BAG = \frac{\pi}{4}(ab - (a-2i)(b-2i)) \qquad [1a]$$

In figure 1b, the elipses are assumed to be symmetric with respect to the center of the tree, with growth being greater along the major axis. Basal area growth can be computed from equation 1b using two cores, one taken along each axis.

$$BAG = \frac{\pi}{4}(ab - (a-2i_a)(b-2i_b)) \qquad [1b]$$

In figure 1c, the elipses are assumed to be asymmetric with resect to the center of the tree, and since the degree to which the inner elipse is off center will not generally be known, there does not exist an obvious geometric method of computing basal area growth from two increment cores. However, if we assume that the ratio of the major and minor axes is proportional to the ratio of the growth along those axes (that the stem does not change its shape as it grows during the increment period) then basal area growth can be computed using equation 1c from two cores taken from opposite sides of the tree along the major axis.(Note: i_1 and i_5 refer to the position of the core around the bole--see figure 2)

$$BAG = \frac{\pi}{4}(ab - (a - (i_1+i_5))(b - (\frac{b}{a}(i_1+i_5))) \qquad [1c]$$

These procedures will provide exact measures of tree basal area growth when the stem is a true elipse and one of the models, 1a,b or c, applies.

D-Tape

The procedure of using a D-tape to measure stem size assumes that the stem is circular. When the stem is not circular, an overestimate of the basal area will result. The overestimate depends on the eccentricity (the degree to which the tree is out-of-round) of the stem.

1/ Basal area growth also includes bark growth. In actual practice these formulas would be modified by a function of the bark factor for the tree, in order to take into account the growth of the tree's bark. To simplify analysis the modifier was ignored in this analysis.

The D-tape measures the circumference of a tree and converts it to a diameter. The cross sectional area of the tree, in terms of the circumference, is given by the formula:

$$A = \frac{C^2}{4 \pi X 144}$$

The circumference of an elipse is approximated by

$$C = \pi \sqrt{\frac{(a^2 + b^2)}{2}}$$

thus the area of an elipse measured with a D-tape is

$$A = \frac{\pi}{4 X 144} \frac{(a^2 + b^2)}{2}$$

The ratio of the area of the elipse measured with a D-tape to the true area is given by:

$$\frac{\frac{\pi}{4 X 144} \frac{(a^2 + b^2)}{2}}{\frac{\pi}{4 X 144} ab} = \frac{\frac{(a^2 + b^2)}{2}}{ab}$$

Hence the ratio of the area computed from D-tape measurements to the area computed from caliper measurements is equal to the ratio of the square of the quadratic mean diameter of the major and minor axes to the square of the geometric mean diameter of the axes. Since, for an elipse, this ratio is always greater than 1, the use of a D-tape on an out-of-round tree will lead to an overestimate of the true basal area. Furthermore, the greater the difference between the lengths of the major and minor axis the greater the percent of overestimate.

Since one cannot determine the major and minor axes of an eliptical stem with a D-tape, there is no exact geometric method for obtaining the basal area growth of an out-of-round tree when a D-tape and an increment core are used for the calculation.

Data Collection

DBH cross sections were available from trees felled in the course of stem analysis by members of the Northern California Forest Yield Cooperative. Forty five out-of-round sections were used in the study. Tracings were made of the last ten year growth period. From these tracings the following were measured: the major and minor axis, the circumference, and 8 increments around the perimeter (see figure 2). The area within the growth period was measured with a planimeter. In measuring the circumference of the section a line was drawn over indentations in the section to simulate the measure that would be obtained using a D-tape (see dotted line in figure 2). The direction of the increments was toward the geometric center of the tree (not necessarily the biological center) since one would not know the location of the pith when using an increment borer.

The ratio of the major axis to the minor axis of the sections ranged from 1.02 to 1.29 (the major axis 2% to 29% longer than the minor axis). The average ratio was 1.11.

Data Analysis

Caliper

The analysis was designed primarily to test for confirmation of the geometric models, but also to determine which of the increments, or combinations of two increments, in conjunction with equations 1a, b, or c, resulted in basal area figures closest to the planimetered area.

Basal area growth was computed from measurements of the major and minor axes of the inner and outer elipses for each tracing. Growth was also computed using the major and minor axes of the outer elipse and the following three combinations of increments. Firstly, increments 1, 3, 5, and 7 were each used to compute basal area growth by equation 1a. Secondly, increment pairs 1 and 3, 1 and 7, 5 and 3, and 5 and 7 were used to compute basal area growth by equation 1b. And finally the sum of increments 1 and 5 was used to compute growth from equation 1c.

To check the validity of the models shown in figure 1, the ratio of the basal area growth computed using increments to the planimetered basal area growth was calculated.

The results of these computations, shown in Table 1, will be discussed below.

D-Tape

Basal area growth was calculated from the circumferences of the inner and outer elipses to simulate the use of a D-tape in measurements of trees on permanent plots. Growth was also calculated using the circumference of the outer elipse and functions of the 8 increments. Since, in this case, there was no function suggested by the geometric model when two increments were used to compute basal area growth the arithmetic mean and the geometric mean were the functions used. More complicated functions could have been investigated but the arithmetic mean was found to be adequate. The ratios of these growth estimates to the planimetered basal area growth were also calculated.

Results

Tables 1 and 2 show the average ratios and some distributional information for the analysis of growth calculation methods using caliper and D-tape respectively. In measuring the sections, increment 1 was always the shorter increment and increment 5 the longer increment along the major axis. But there were no design differences in the experiment with respect to increments 3 and 7. Therefor, results involving increments 1 and 5 are differentated in Table 2, but those involving 3 and 7 are combined.

TABLE 1

Ratio of computed basal area growth to planimetered basal area growth when calipers are used to measure stem size

	Model 1a					Model 1b		Model 1c	
Increment(s)	1	3	5	7	$\frac{(3+7)}{2}$	1,3 1,7	5,3 5,7	1+5	Periodic measures with calipers
Mean	.983	1.014	1.392	.973	.996	.994	1.135	1.143	1.096
Standard Dev.	.1941	.1701	.1760	.1683	.0970	.1251	.1125	.0530	.0594
Percent of sections within specified range of planimetered value.*									
± 5%	20%	31%	-	31%	36%	39%	-	-	-
±10%	40%	56%	-	58%	78%	53%	-	-	-
±15%	59%	69%	-	73%	91%	90%	-	-	-

* Values less than 5% are not reported.

TABLE 2

Ratio of computed basal area growth to planimetered basal area growth when DBH tape is used to measure stem size

Increment(s)	1	3 7	(1+5)/2	(1+3)/2 (1+7)/2	(3+7)/2	(2+6)/2 (4+8)/2	(3+5)/2 (7+5)/2	(2+3)/2	Periodic measures with Dbh tape.
Mean	.993	1.000	1.204	1.001	1.002	1.065	1.211	.954	1.096
Standard Dev.	.1990	.1630	.073	.1290	.097	.0390	.1155	.1150	.093
Percent of sections within the specified range of planimetered.*									
+5%	20%	27%	-	32%	23%	-	-	-	-
+10%	40%	54%	-	56%	76%	-	-	-	-
+15%	67%	72%	-	82%	95%	-	-	-	-

* Values less than 5% are not shown.

Caliper

In model 1a, growth does not vary around the stem. So, (using equation 1a), increments 1, 3, 5, and 7 should give similar basal area growth figures. Furthermore, the ratio of growth computed using equation 1a to planimetered growth should be equal to 1.

While calculations based on increments 1, 3, and 7 were similar, calculations based on increment 5 resulted in an overestimate of 39 percent. We cannot conclude that increment does not vary significantly around the stem; or that any randomly chosen single increment along the major or minor axis will be appropriate for computing basal area growth.

In model 1b, increment does vary around the stem, but is symetric around the center. Model 1b presumes that basal area growth computations from equation 1b involving increments 1 and 3, and 1 and 7, will be similar to those involving increments 5 and 3, and 5 and 7. That is, when two increments are taken at right angles (one along the major axis and one along the minor axis), it does not matter which end of the major axis is used (ie. uphill or downhill). However, there is a 19 percent difference between calculations involving increment 1 and those involving increment 5.

Finally, in model 1c, the assumption was made that, although the elipses were not symetric around the center of the figure, the ratios of the increments along the major and minor axes were equal to the ratio of the axes ($i_a/i_b = a/b$). If this is true the ratio of computed growth to planimetered growth should be close to 1. However, there was a 14 percent difference between these methods of basal area growth computation. Since this difference is statistically significant ($\alpha = .05$), we conclude that the ratio of the growth along the major and minor axes is not proportional to the ratio of the major and minor axes.

None of the above models could be confirmed by the data. Part of the reason for this lies in the fact that the basic assumption of elipticity is incorrect. On the average, calipers overestimated the basal area growth of these trees by almost 10 percent. However, the use of some of the increments, in conjunction with current size as measured by calipers, gave basal area growth figures that were consistently close to the planimetered values. The use of increments 1, 3, 7, or the average of 3 and 7 in equation 1a, and the use of increments 1 and 3, or 1 and 7 with equation 1b gave basal area growth figures close to the planimetered growth. The overestimate of tree basal area using calipers was compensated by an under estimate of the average increment.

As has been stated, many combinations of radial increments can be used to compute basal area growth. In this study the best results were obtained by using the average of two increments taken along the minor axis (increments 3 and 7) in conjunction with equation 1a. Computing basal area growth in this manner resulted in figures within 10% of planimetered basal area growth better than 75% of the time.

When the use of only one increment core was tested, the use of an increment from the minor axis again resulted in the best estimate of basal area growth. Therefor, when calipers are used to measure tree size, and one or two increments are taken to compute basal area growth, it is recommended that the increment(s) be taken along the minor axis.

D-Tape

A similar situation occurs when using D-tape measurements. A D-tape overestimates the basal area of an eccentric stem, but in the computation of basal area growth this overestimate is compensated by the use of an increment that is less than the average increment (such as increments 1, 3, or 7, or combinations of these increments).

Again, it is recommended that one or two increments along the minor axis be used when computing basal area growth. The use of increments along the minor axis resulted in basal area growth figures as good as any other combination of increments, and avoids the uncertainty of determining which end of the major axis is increment 1 and which is increment 5.

Finally whether using calipers or a D-tape, boring out-of-round trees along the minor axis appears to give the best estimate of basal area growth.

Part Two: Trees with Circular Stems

Since crown asymmetry is associated with within-tree variation in diameter growth (Gray, 1956), a test was designed to determine if the within-tree variation in diameter growth could be explained by the relative height of the crown around the tree.

This portion of the analysis was completed before stem analysis sections became available, and thus the true basal area growth of the tree is not known. Instead we assume that the true basal area growth is close to that calculated from an average of the four increments taken around the tree.

Data Collection

A total of 105 trees were bored on 17 plots on Forest Service land near Meadow Valley, California. No obviously out-of-round trees were chosen. The plots were mapped and DBH, total height, and heights to the lowest and highest points of the crown base (low crown height and high crown height) were recorded. Each tree was bored 4 times at breast height: directly beneath the lowest and highest crown heights, and directly beneath two points of average crown height on either side of the tree. The azimuth of each boring was also recorded.

Analysis

The analysis of the effect of crown length on within-tree radial growth is complicated by differences in the growth of individual trees. The site, the position of the tree in the canopy, and the tree's genetic growth potential are some of the factors that confound the analysis. In order to deal with this, an analysis of variance model was proposed.

$$Y_{ij} = \mu + \alpha_i + \beta_j$$

Where

Y_{ij} = radial growth of crown length class (j) of tree (i)

μ = overall mean

α_i = the effect of tree (i) (i.e. tree-to-tree variation)

β_j = the effect of crown length class (j) (i.e. within-tree variation)

For each tree, radial growth was classified according to the relative crown length directly above the boring, so that each tree had one radial growth measurement in the short crown length class, one in the long crown length class, and two in the average crown length classes. An analysis of variance was performed in which the total variance was

Table 3. Class Means and Anova Table for Radial Growth Study

Class Means

Crown Length Class	Long	Average1	Average2	Short
Mean (mm)	11.30	10.25	10.52	9.39

ANOVA

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	
Between Trees	11190.75	104	107.60	
Explained by Crown Class	119.28	3	39.76	
Residual	2464.97	312	7.90	$= \sigma^2$
Total	13775.00	419		

$$\text{F-statistic} = 39.76/7.90 = 5.03 > F_{(.05;\ 3,312)} = 2.60$$

partitioned into (1) the variance between trees, (2) the within tree variance explained by crown length class (short, average1, average2, long), and (3) residual variance (Table 3).

Results

Table 3 shows the class means and the analysis of variance table for the model above. After accounting for between tree effects, crown length class was found to be a statistically significant factor (α = .05) in the analysis of variance model. However, crown length class accounted for only 5 per cent of the within tree variance.

The average difference between the increments corresponding to short and long crown length classes was 1.4mm (about 14% of the mean 5-year growth increment). This difference was statistically significant (α = .05). Further, the difference increased as asymmetry of the tree crown increased. When the difference between the long and short side of the crown was greater than or equal to 25 feet, the difference between the corresponding increments was 20% of the mean. And when the long side of the crown was at least 30 feet greater than the short side of the crown, the difference between the increments was 25% of the mean. These differences were also statistically significant (α = .05).

Summary and Conclusions

None of the eliptical models in Part 1 completely described the shape of the eccentric tree cross-sections tested. And calipers and D-tape both resulted in an overestimate of cross sectional basal area. However, when computing basal area growth this overestimation can be compensated for by a judicious choice of the position from which the trees are bored for radial increment. The use of increments along the minor axis of the tree (increments 3 and 7 in figure 2) is recommended for use with both calipers and D-tape.

It should be noted that although basal area growth will be more accurately estimated, growth percent will be underestimated. This is because the use of calipers or D-tape result in an overestimate of the basal area of out-of-round trees.

For trees that are not out of round, this analysis indicates that relative crown length can be used to slightly improve estimation of radial growth. To acheive this improvement the tree should be bored beneath a point of average crown length. Since there are, in principle, at least two such points for each tree, the cruiser can follow this rule and still find a convenient place from which to bore the tree.

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