CO-OP REDWOOD YIELD RESEARCH PROJECT
Department of Forestry and Conservation
College of Natural Resources
University of California
Berkeley, California 94720

Lee C. Wensel
Associate Professor

Bruce Krumland
Assistant Specialist

Research Note No.7 October 1, 1977

Variable Density Yield Equations for Natural Stands of Coastal Conifers

by

Bruce Krumland and Lee C. Wensel

Abstract

Yield equations are developed for uncut young growth stands twenty years breast high age and greater in which the basal area stocking of stems 11.0 inches DBH and larger is at least 90 percent conifers. The equations use current breast high age, site index, basal area, and percent basal area in species other than redwood as access points. The predictions are for board or cubic foot volume at any future age up to 100 years of breast high age.

These yield equations are considered peripheral to the major focus of this cooperative as the stand conditions to which they are applicable are restricted and there is a certain rigidity in definitions that must be adhered to in order to obtain reliable predictions. The models in this note were developed at the request of several cooperators and for the supplementary means they provide to check the accuracy and reliability of tree models.

There is considerable room for refinement in these models. If sufficient response and feedback are received, they will be updated in the future.

I Introduction

Our current research efforts are focusing on the individual tree approach to stand growth and yield estimation. This approach has been found to be necessary when growth estimates are needed for stands that do not approach the classical definition of even-aged monocultures, that are characterized by variable structure, or have been cutover. Several cooperators have indicated however that a simple stand model that can be used to provide yield predictions for variable density stands of natural origin composed of mixtures of conifers would have immediate use. Also, the currently available data base is marginally adequate to cover the entire range of stand and management conditions for which information is desired for. Hence, a stand model can be used to check the logic of tree models under limited conditions.

The remainder of this report describes the development of a stand model that is applicable only to stand components 11.0 inches DBH and larger and is restricted to uncut young growth stands of natural origin which are composed of at least 90 percent conifers by basal area.

II Model Development

Sullivan and Clutter (1972) presented a model for Loblolly Pine that is simultaneously a growth and yield model. The model is a system of equations that uses current age, site index, and basal area to predict current volume and volume at any other future age. These models can be written as

Yield Model - EQ.1

$$E(\ln V_1) = b_0 + b_1 S + b_2 A_1^{-1} + b_3 \ln B_1$$

Growth Model - EQ.2

$$E(\ln V_2) = b_0 + b_1 S + b_2 A_1^{-1} + b_3 (A_1/A_2) \ln B_1 + b_4 (1-A_1/A_2) + b_5 (1-A_1/A_2) S$$

where

S = site index
A_i = stand age in years at the i^{th} measurement
B_1 = basal area at measurement 1

V_i = volume at the i^{th} measurement
b_j = parameter estimates
E^j denotes expected value
ln denotes the natural logarithm

These models are considered desirable because the form implies relationships that conform with agreed upon concepts of stand development. Also, if $A_1 = A_2$ in EQ.2, then the growth model reduces to the yield model. Hence, the models are considered to be 'compatible'.

Equation 2 was used as a starting point in developing a model for coastal stands and was subsequently modified to incorporate differences in species composition.

III Data

The data utilized in this study came from the records of 159 permanent growth plots maintained by members of the cooperative in Del Norte, Humboldt, and Mendocino counties. Plots ranged in size between one tenth and one half acre. The plots were all located in apparently evenaged stands of natural origin and had not experienced any partial harvests. Plots with more than 10 percent of the basal area in hardwood stems 11.0 inches DBH and larger or stands less than 20 years of age at breast height were not included. Two measurements on each of these plots were selected at random; the initial measurement is denoted by '1' and the terminal measurement by '2'. For each measurement set 'i', the following items were computed on a per acre basis.

B_i = basal area in square feet in stems 11.0 inches DBH and larger at measurement 'i'

P_i = percent basal area in species other than redwood. Douglas fir averaged 91 percent of this component.

VB_i = Board foot scribner volume of all stems 11.0 inches DBH and larger in thousands of board feet.

VC_i = Cubic foot volume of all stems 11.0 inches DBH and larger in thousands of cubic feet

SR = redwood site index - height in feet at 50 years

SD = Douglas fir site index - height in feet at 50 years

AR_i = average breast high age of redwood site trees

AD_i = average breast high age of Douglas fir site trees

The means and standard deviations of the sample growth plot data are shown in Table 1. The two measurement records on a given plot comprise an observation set for subsequent analysis. The way in which the data were summarized implicitly incorporates components for ingrowth into the 11.0 inch size class and for mortality. The volume equations used for summarizing plot volumes were those derived by Krumland, Dye, and Wensel(1977a). In instances where total heights were missing for individual trees, height-diameter curves were fitted to each plot to provide access points for volume equations. Douglas fir volume equations were used for all 'other species'.

For redwood site index, the curves developed by Krumland and Wensel(1977b) were used. For Douglas fir, King's(1967) were used. In cases where site index for redwood or Douglas fir was unavailable, they were estimated by the procedures described by Krumland and Wensel(1977c). Where average breast high age of redwood or Douglas fir was unavailable, it was estimated by one the following equations

$$AD_i = -10.6 + .0435(SR) + 1.037(AR_i)$$

$$R^2 = .86$$

$$s_{y.x} = 5.7$$

$$AR_i = 7.13 + .037(SD) + .84(AD_i)$$

$$R^2 = .86$$

$$s_{y.x} = 5.1$$

A description of the sample data used to derive these equations can be found in Krumland and Wensel(1977c).

IV Analysis

The sample data were initially fitted to EQ.2 by linear least squares for both cubic and board foot volumes using redwood site index and age. Species composition was not initially considered. Multiple correlation coefficients were .93 and .92 respectively which indicated the model form was adequate. The term "b_5" when tested, was statistically insignificant in both cases and was subsequently dropped from the model

Next, the residuals (difference between actual and predicted values) were machine plotted against percent basal area in other species at the initial measurement. It was readily apparent that as the species composition shifted from redwood to other conifers, the estimates of future yield increased. This relationship was anticipated for

several reasons: (1) when redwood and Douglas fir occur in mixture, site index determined for Douglas fir is usually about twenty feet greater than redwood, (2) in stands above twenty to thirty years of age, Douglas fir occupy dominant positions and are usually taller than associated redwoods, and (3) redwood tends to taper more and have a greater bark thickness than Douglas fir. Hence, for conifers of the same DBH and total height, less volume is found in redwood.

Several attempts were made to modify EQ.2 to account for the interactions of species composition at a given age and basal area on future yields. Results were inconclusive after anything more than a simple proportionate term containing percent basal area of other species was added to the model. The model was subsequently redefined as

$$E(\ln V_2) = b_0 + b_1 S + b_2 A_2^{-1} + b_3 (A_1/A_2) \ln B_1 + b_4 (1 - A_1/A_2) + b_5 \ln(P_1 + .05) \qquad \text{EQ.3}$$

Estimated coefficients for this model (b_i) were derived for four different cases; cubic and board foot volumes using redwood based site and age amd the same two volume measures using Douglas fir based site index and age. Estimated coefficients are shown in tables 2 and 3.

V. Estimating Future Yields

These equations can be used to predict future volume yields for specific stands if current basal area, age, site index, and species composition have been determined. For example, if the following items have been determined

$SR_1 = 100$
$AR_1 = 30$
$P_1 = .33$
$B_1 = 175$

and redwood based equations are used to predict cubic yields at age 60(A_2), then

$$E(\ln VC_2) = -3.88 + .0054(100) - 26.8/60 + 1.11(20/60)\ln(175) + 7.41(1-20/60) + .134(\ln(.33+.05))$$

$$= 2.93$$

$$VC_2 = e^{2.93} = 2.71828^{2.93} = 18.8 \text{ M cubic feet}$$

Table 1. Per acre means and standard deviations of 159 sample growth plots

	Mean	Standard Deviations
Redwood site index (SR)	108	15.6
Initial redwood age (AR_1)	47	13
Terminal redwood age (AR_2)	56	13
Douglas fir site index (SD)	133	13.5
Initial doug-fir age (AD_1)	40	13
Terminal doug-fir age (AD_2)	49	13
Initial basal area (B_1)	259	150
Initial percent other species (P_1)	.35 *	.29
Initial cubic volume (VC_1)	8.4	6.0
Terminal cubic volume (VC_2)	11.0	7.0
Initial bd-ft volume (VB_1)	46.9	38.0
Terminal bd-ft volume (VB_2)	63.4	46.5

* species composition percentages ranged from 0 to 100

Table 2. Redwood age and site index based growth and yield equation coefficients

	b_0	b_1	b_2	b_3	b_4	b_5	R^2	$S_{y \cdot x}$ *
Cubic Volume	-3.88	.0054	-26.8	1.11	7.41	.134	.96	15%
Bd-ft Volume	-2.69	.0067	-33.4	1.21	8.05	.163	.96	18%

Table 3. Douglas Fir age and site index based growth and yield equation coefficients

	b_0	b_1	b_2	b_3	b_4	b_5	R^2	$S_{y \cdot x}$ *
Cubic Volume	-4.75	.0066	-19.0	1.19	7.75	.138	.96	14%
Bd-ft Volume	-3.74	.0081	-23.3	1.31	8.50	.168	.96	17%

* Standard deviation of residuals are expressed as a percentage in normal volume units

Conventional yield tables can be generated from these equations simply by substituting various combinations of the independent variables. Some examples with redwood based equations are shown in Appendix I. It should be noted that if several sample points have been taken in a stand to determine age, site index, basal area, and species composition, predictions should be made for each sample point and then averaged to obtain an estimate of future yield for the stand.

VII. Discussion and Summary

The yield equations developed in this report are essentially for limited stand conditions and components. There are several possibilities for refinement within this general system of models that can increase their usefulness and precision. For example, it may be desirable to provide estimates based on stand components down to much smaller diameter limits. Incorporating additional observations may provide enough information to distinguish 'optimum' density and species composition levels although the limited analysis performed in the course of this study indicated the models would rapidly grow in complexity if this were undertaken (see Progress Report 2 for example). Lastly, it is recognized that in applications, volume is usually determined for sample plots along with site index, age, and the other independent variables needed for yield predictions. A limited bivariate analysis indicated that standing volume at the initial measurement can be incorporated into these models and results in a reduction of over 100 percent in the residual variance. (As a tentative rule of thumb, predicted volume at some future age could be multiplied by the ratio of measured to predicted volume at the current measurement for a 'local correction'). Many of these items can be considered in an update of this report if enough interest is expressed in these kinds of models.

Literature Cited

King, James E. 1966. Site index curves for Douglas fir in the Pacific Northwest. Weyerhaeuser For. Rep. No.8 Weyerhaeuser For. Res. Cntr. Centralia, Wash. 49p.

Krumland, B., L. C. Wensel, and J. B. Dye, 1977a. Young Growth Volume Tables for Coastal California Conifers. Coop Redwood Yield Res. Proj., Res. Note No. 3. U.C. Berkeley, mimeo.

Krumland, Bruce and Lee C. Wensel, 1977c. Procedures for Estimating Redwood and Douglas fir Site Indexes in the North Coastal Region of California. Coop Redwood Yield Research Proj., Res. Note No. 5, U.C. Berkeley, mimeo

Krumland, Bruce and Lee C. Wensel. 1977b. Height growth patterns and fifty year base age site index curves for young-growth coastal redwood. Res. Note No.4. Coop. Redwood Yield Research Proj. U.C. Berkeley. Mimeo.

Sullivan, Alfred D. and Jerome L. Clutter, 1972. A Simultaneous Growth and Yield Model for Loblolly Pine. For Sci 18:1. p76-86.

Appendix I

Redwood age and site index based board foot yield tables. Volumes are in thousands of board feet Scribner scale.

Site Index = 80

Basal area at age 30 = 100 square feet

B.H. Age	Percent Redwood Basal Area				
	100	75	50	25	0
30	5.9	8.0	8.8	9.3	9.8
40	14.6	19.6	21.6	23.0	24.0
50	25.1	33.6	37.1	39.4	41.2
60	36.0	48.1	53.1	56.5	59.0
70	46.5	62.3	68.7	73.0	76.3
80	56.4	75.5	83.3	88.6	92.6
90	65.6	87.7	96.8	102.9	107.6
100	73.9	98.9	109.2	116.0	121.3

Basal area at age 30 = 150 square feet

B.H. Age	Percent Redwood Basal Area				
	100	75	50	25	0
30	9.7	13.0	14.3	15.2	15.9
40	21.1	28.2	31.2	33.1	34.6
50	33.7	45.0	49.7	52.8	55.2
60	45.9	61.5	67.9	72.1	75.4
70	57.4	76.8	84.7	90.1	94.1
80	67.8	90.7	100.1	106.4	111.2
90	77.2	103.3	114.0	121.1	126.6
100	85.6	114.6	126.4	134.4	140.4

Basal area at age 30 = 200 square feet

B.H. Age	Percent Redwood Basal Area				
	100	75	50	25	0
30	13.7	18.4	20.3	21.5	22.5
40	27.4	36.6	40.4	43.0	44.9
50	41.5	55.5	61.2	65.1	68.0
60	54.7	73.1	80.7	85.8	89.7
70	66.6	89.1	98.3	104.5	109.2
80	77.2	103.3	114.0	121.2	126.7
90	86.6	115.9	127.9	136.0	142.1
100	95.0	127.1	140.3	149.1	155.8

Site Index = 100

Basal area at age 30 = 100 square feet

B.H. Age	Percent Redwood Basal Area				
	100	75	50	25	0
30	6.8	9.1	10.0	10.6	11.1
40	16.7	22.3	24.6	26.2	27.4
50	28.6	38.3	42.3	44.9	46.9
60	41.0	54.9	60.6	64.4	67.3
70	53.0	71.0	78.3	83.3	87.0
80	64.3	86.1	95.0	101.0	105.5
90	74.7	100.0	110.4	117.3	122.6
100	84.3	112.8	124.5	132.3	138.3

Basal area at age 30 = 150 square feet

B.H. Age	Percent Redwood Basal Area				
	100	75	50	25	0
30	11.1	14.8	16.3	17.3	18.1
40	24.1	32.2	35.5	37.8	39.5
50	38.4	51.3	56.7	60.2	62.9
60	52.4	70.1	77.3	82.2	85.9
70	65.4	87.5	96.6	102.7	107.3
80	77.3	103.4	114.1	121.3	126.8
90	88.0	117.7	129.9	138.1	144.3
100	97.6	130.6	144.1	153.2	160.1

Basal area at age 30 = 200 square feet

B.H. Age	Percent Redwood Basal Area				
	100	75	50	25	0
30	15.6	20.9	23.1	24.5	25.7
40	31.2	41.8	46.1	49.0	51.2
50	47.3	63.2	69.8	74.2	77.5
60	62.3	83.4	92.0	97.8	102.2
70	75.9	101.6	112.1	119.1	124.5
80	88.0	117.8	130.0	138.1	144.4
90	98.8	132.2	145.8	155.0	162.0
100	108.3	144.9	159.9	170.0	177.7

Site Index = 120

Basal area at age 30 = 100 square feet

B.H. Age	Percent Redwood Basal Area				
	100	75	50	25	0
30	7.7	10.3	11.4	12.1	12.7
40	19.0	25.4	28.1	29.8	31.2
50	32.6	43.7	48.2	51.2	53.5
60	46.8	62.6	69.0	73.4	76.7
70	60.5	80.9	89.3	94.9	99.2
80	73.3	98.1	108.3	115.1	120.3
90	85.2	114.0	125.8	133.7	139.8
100	96.1	128.6	141.9	150.8	157.6

Basal area at age 30 = 150 square feet

B.H. Age	Percent Redwood Basal Area				
	100	75	50	25	0
30	12.6	16.9	18.6	19.8	20.7
40	27.4	36.7	40.5	43.0	45.0
50	43.7	58.5	64.6	68.7	71.8
60	59.7	79.9	88.2	93.7	97.9
70	74.6	99.8	110.1	117.0	122.3
80	88.1	117.9	130.1	138.3	144.5
90	100.3	134.2	148.1	157.4	164.5
100	111.2	148.9	164.3	174.6	182.5

Basal area at age 30 = 200 square feet

B.H. Age	Percent Redwood Basal Area				
	100	75	50	25	0
30	17.8	23.9	26.3	28.0	29.2
40	35.6	47.6	52.5	55.8	58.4
50	53.9	72.1	79.5	84.5	88.4
60	71.0	95.0	104.9	111.5	116.5
70	86.5	115.8	127.8	135.8	141.9
80	100.3	134.3	148.2	157.5	164.6
90	112.6	150.7	166.3	176.7	184.7
100	123.5	165.2	182.3	193.8	202.5