



CO-OP REDWOOD YIELD RESEARCH PROJECT

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Research Note No. 8

Generalized Height Diameter Equations
for Coastal Conifers

by

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ABSTRACT

Models are presented which can be used to predict the total height of trees on growth and inventory plots. These estimators require only the computation of average DBH and average total height of a sample of trees selected from the upper one fifth of the stand (plot) diameter distribution. This stand component is also used to estimate site index so the procedure is efficient. With small samples, these equations were found to be very accurate in local volume table construction.

I. INTRODUCTION

Height-diameter curves have long been used in forest growth and inventory analysis as a means of predicting missing total height data. As a general method, a subsample of trees in a stand or plot are selected for height measurement in addition to DBH. Trees are segregated by species and a species-dependent curve is prepared by graphical techniques or coefficients in a height-DBH model which are estimated by regression techniques.

In practice, graphical techniques are time-consuming and are not used much. Regression techniques are less time-consuming particularly when desk top or batch processing computers are available. Experience has shown however that small samples (less than 16 trees/plot) often result in ill-conditioned regression estimators which give poor predictions of height particularly in the tails of the diameter distribution. As most of the stand volume is in the upper tail of the diameter distribution, this situation can have a significant impact on volume estimates.

Some simple and consistent method of estimating heights is also needed for use with growth prediction models. In situations where little more is known other than site index and age, some means of generating an entire height distribution is also needed.

This research note describes the development of two general height diameter models which provide satisfactory solutions to these criticisms and needs. The models are simple to use and require only the computation of two simple averages to be accessed.

II. MODEL DEVELOPMENT

Curtis (1967) suggested that the height-diameter model

$$H = \exp[a + b/D^c] \quad (I)$$

where

H = total height

D = DBH

$\exp(x)$ = 2.7185 raised to the power of x

a, b, c = model parameters

would be a generally satisfactory model to use in height prediction on inventory and growth plots. Experimentation has shown that the parameter c varies with dominant height or age. Curtis (1967) has reported a similar observation.

If one point is known on the curve represented by

equation I with coordinates say H_m and D_m , then the equation will reduce to

$$H = (H_m) \exp[b(D^c - D_m^c)] \quad (II)$$

In this form, predicted height will equal H_m when D equals D_m .

Choice of H_m and D_m .

As equation II is conditioned to go through the point $[H_m, D_m]$ it would seem logical to define these variables in terms^m of dominant trees as this would center the height predictions in the stand component where most of the volume is. Secondly, as site trees are measured for total height anyway, defining H_m and D_m as average height and diameter of trees suitable for^m site index estimation is both efficient and satisfactory.

III. DATA

Trees from single measurements on 284 permanent growth plots in young growth stands were used as observations for subsequent analysis. On plots with multiple measurement sets, only one set was used. These plots were located on apparently evenaged stands. About 30% had been thinned. On each plot, the following procedure was used for each of the following species groups; Redwood, Douglas fir, and other whitewoods. For each group, if four or more trees suitable as site trees were measured for height, H_m and D_m were computed. Otherwise the plot was not used^m for the particular species. Three to five trees of the same species with measured heights were then randomly chosen as observations for model fitting.

This procedure produced three species-dependent data sets. Initial analysis of covariance indicated that Douglas fir and other whitewoods could be combined into one set (called Douglas-fir). Differences between redwood data and this combined set were significant so separate models were developed for each.

Restricted stand conditions

Another initial analysis of covariance was made to see if there were any differences between height diameter relationships in cutover and uncut stands. The basic result of this analysis is the models described in the next section are not applicable to stands that have been "severely

^{1/} For both redwood and Douglas fir, suitable site trees consist of the 20 percent largest trees by DBH for each species.

thinned from above". In other words, if partial harvesting has removed most of the dominant and codominant trees, these models will not give reliable results. There were no significant differences between uncut stands and stands that had been moderately thinned from below or were thinned for crop tree spacing. Thirty eight plots were from stands which had much of the main canopy removed and the corresponding sample trees were deleted from subsequent analysis.

IV. ANALYSIS

Several variations of equation II were initially postulated, each having the parameters "b" and "c" being represented by different functions of H_m and or D_m . Tests of performance (see section VIII) and statistical inference²⁷ led to the following model:

MODEL I - Exponential Form

$$H = (H_m) \exp[b(D_m^{c_0+c_1H_m} - D_m^{c_0+c_1H_m})]$$

Coefficient estimates and summary statistics for redwood and Douglas fir models are shown in Table 1. Forcing the model to go through the point (H_m, D_m) introduces a possible bias that is similar to the one encountered by averaging age and height in estimating site index rather than averaging individual site index estimates. A test for bias was made by adding a multiplicative parameter (a_0) to the model. This produced an estimate of $a_0=1.002$, standard error = .015 which was considered insignificant. Hence, the model was left as is.

V. ALTERNATIVE MODEL

An alternative model with different properties was also developed to provide an additional means of comparison. This model has the form

Model II - Logistic Form

$$H = a_0 H_m / (1. + \exp[a_1 + a_2(D/D_m) + a_3(DH_m/D_m)])$$

Plotting H/H_m against D/D_m indicated that this form would be appropriate. Values of the coefficients and a statistical summary are shown in Table 2.

Model II has the property that predictions are always positive and will never exceed a proportion (a_0) of H_m .

²⁷ Coefficients were estimated by nonlinear least squares. Tests of inference were based on likelihood ratio tests proposed by Gallant (1975).

Table 1

Model I - Exponential Model

Coefficients and Statistical Summary for Redwood and Douglas-Fir

	b	c ₀	c ₁	R ²	standard deviation	sample size
Redwood	-4.47	-.105	-.0016	.89	11.4 ft.	631
Douglas-fir	-2.52	-.240	-.0018	.98	6.0 ft.	819

*Table 2

Model II - Logistic Model

Coefficients And Statistical Summary For Redwood And Douglas-Fir

	a ₀	a ₁	a ₂	a ₃	R ²	standard deviation	sample size
Redwood	1.07	1.67	-4.3	.0004	.89	11.3 ft.	631
Douglas-fir	1.06	.84	-2.75	-.008	.98	6.0 ft.	819

VI. MODEL COMPARISONS

Both models give virtually the same predictions of total height given DBH for various combinations of H_m and D_m . Differences occur mainly in the tails of the diameter distribution. Model II gives almost constant predictions of height for trees with DBH around D_m and larger. This is in conformance with the observation that heights of dominant trees in even-aged stands are fairly constant and independent of DBH. Model I gives increasing predictions of height for increases in DBH. Thus, in some exceptional instances where the variation of dominant tree DBH is large, Model I may overestimate heights for a few trees.

In contrast, Model I produces height estimates for small trees (1.0 inches DBH) that are reasonable regardless of H_m and D_m . As H_m and D_m increase, Model II produces estimates of height for a 1.0 inch tree which become so large they are sometimes illogical. Attempts to constrain Model II to produce a prediction of 4.5 feet for a tree of zero inches DBH indicated a more complex model would be necessary to correct for distortions in height prediction for larger trees.

As the faults of these models occur in rather rare situations, there is little evidence to recommend one over the other. Both appear to be satisfactory for general use.

VII. SAMPLING SIMULATIONS

How these models compare with alternative methods available will depend on the form of the local plot height-diameter equation chosen, the sampling rule, and the proportion of trees subsampled for height. Curtis (1967) reported on 13 local models, all of which were apparently satisfactory. Hence, one model

$$H = a + b \log D$$

was chosen as a local model for subsequent analysis.

Fifty-four plots with all trees measured for height and diameter were available for empirical comparisons. These plots were not used in model development, and were comprised mostly of conifers. About one-third of the plots had been moderately thinned. There were a sufficient number of plots so the Douglas fir test cases could be divided into two age classes; less than 20 years breast height age and over 20.

Seven possible sampling schemes were postulated for comparative purposes. Sampling simulations were done for each species group separately. Plots were selected if there were 15 trees or more of a given species group present.

- 1) Model I and II -- If the largest 20% of stems consisted of three to eight trees, all of them were used to compute H_m and D_m . Otherwise, a random sample of eight trees from the largest 20% were selected.
- 2) All -- the local model was fit to all trees of a given species group on the plot
- 3) 15% S -- Trees were sorted by DBH and approximately 15% were systematically selected for local model fit (at least 4 were selected).
- 4) 35% S -- Same as 3 only 35% of the trees were selected.
- 5) 15% R -- Random sample of 15% of the trees (at least four trees were selected).
- 6) 35% R -- Random sample of 35% of the trees.

For each plot, each sampling scheme was used to derive predictions of total height for each tree, schemes 2-6 being based on fitting the sample to the local model. As comparative statistics, root mean square deviations (RMSD) were computed for each scheme on each plot. Average RMSD for each scheme is shown in Table 3. Assuming that sampling scheme 2 (All) is the best we can do, all schemes performed roughly the same. There was some indication, however, not evident in summary statistics, that samples of size 4-12 trees per plot are not large enough to get consistent local height diameter models especially in random samples. On two plot, slope coefficients were negative. On three plots, some trees had negative predicted heights. Small samples do not effect predictions based on Models I and II nearly as much.

A parallel test was made to see how well predicted heights compare with measured heights relative to a standard height diameter cubic volume equation. For all seven sampling schemes, the ratio between plot species volumes based on predicted heights versus actual heights were computed. The average ratios, standard deviations and ranges are shown in Table 4. Models I and II perform very well. The impact of small samples (15% sampling proportions) are much more evident in this comparison, particularly in redwood. Both models I and II performed consistently well presumably because of the conditioning in the upper tail of the diameter distribution where most of volume is located.

Table 3

Average Root Mean Square Deviations For Seven Sampling Schemes

Sampling Scheme	Douglas-Fir 20 yrs +	Douglas-Fir 20 yrs -	Redwood 25 yrs +
Model I	8.96	3.18	10.89
Model II	8.94	3.21	10.23
ALL	7.46	2.74	9.92
15% S	9.72	3.02	10.66
35% S	8.94	2.82	10.47
15% R	9.21	3.09	12.28
35% R	8.81	2.84	10.77
	12 plots	31 plots	22 plots

Table 4

Average Ratios Of Estimated Plot Volumes Based
On Predicted Heights With Actual Heights

Douglas-Fir - 13 plots 30 years and older

Sampling Scheme	Ratio	Standard Deviation	Min	Max
Model I	.99	.017	.96	1.01
Model II	.99	.018	.96	1.02
ALL	1.00	.004	.99	1.01
15% S	1.00	.037	.91	1.04
35% S	1.00	.024	.96	1.04
15% R	.98	.028	.94	1.02
35% R	1.01	.024	.95	1.04

Table 4 (continued)

Redwood - 22 plots 25 years and older

Model I	.99	.017	.95	1.02
Model II	.99	.019	.95	1.02
ALL	1.00	.007	.99	1.02
15% S	1.00	.048	.92	1.07
35% S	1.00	.033	.95	1.06
15% R	.98	.103	.71	1.24
35% R	.99	.037	.92	1.07

VIII. APPLICATIONS

Both model I and II provide a fairly simple and accurate basis for estimating missing heights and subsequent volume estimation on inventory and growth plots. Model I especially can be reduced to a fairly simple form for making a local volume table based on tree diameter alone (see Appendix I.) In situations where no tree heights are available and site index and average breast high age of dominants are known, H_m can be reasonably predicted by site curves.

LITERATURE CITED

- Curtis, Robert O. 1967. Height-diameter and height-diameter-age equations for second-growth Douglas fir. For. Sci:13(4) 365-375
- Gallant, A. R. 1975. The power of the Likelihood Ratio Test of Location in Nonlinear Regression Models. JASA, 70(349) 198-203.

Appendix I

Suggested Procedure for Local Volume Table Construction

I. Assume a volume equation of the form

$$V = a_0 D^{a_1} H^{a_2}$$

is to be used.

II. Estimate H_m and D_m by species from a sample (plot by plot preferably or in an entire stand if it is relatively evenaged and homogenous).

III. Look up in table I to get the coefficients b , c_0 and c_1 for the model

$$H = (H_m) \exp(b(D^{c_0+c_1 H_m} - D_m^{c_0+c_1 H_m}))$$

IV. Compute the diameter exponent t

$$t = c_0 + c_1 H_m$$

V. Compute the access constant q

$$q = (a_0)(H_m^{a_2}) \exp(-(b)(a_2)D_m^t)$$

VI. Compute the product s

$$s = a_2 b$$

VII. The local volume equation is

$$V^x = q D^{a_1} \exp[sD^t]$$