



CO-OP REDWOOD YIELD RESEARCH PROJECT

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YOUNG GROWTH VOLUME TABLES FOR COASTAL CALIFORNIA CONIFERS

by

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The volume tables and equations in this report represent an assimilation of currently available dendrometry and felled tree measurements for coastal redwoods, Douglas fir, and other whitewoods (lowland fir, western hemlock, and sitka spruce). Volumes for other whitewoods are presented as composite tables because sufficient samples were unavailable to generate separate tables and, collectively, they represent only a minor timber component in the region.

These tables were generated for use in concurrent growth and yield studies for the region and will be used as a standard tree volume basis in situations where local volume information is unavailable. These tables may also be used for general inventory work if field checking indicates they are accurate for specific tasks. Procedures for field checking and adjusting these tables are also described.

The volume tables use breast-high diameter outside bark in inches and total height in feet as access points. Scribner volumes are from stump to a six inch top inside bark. Cubic volumes are from stump to the tip of the tree. If desired, cubic equations can be converted to metric units (see Table 1).

All volumes are gross with no allowance for defect or breakage.

Tree Measurements and Sampling Procedures

Dendrometry data - The University's Barr and Stroud optical dendrometer, model FP 15, was used to take measurements of bole height and outside bark stem diameters at selected points along tree stems.

Sampling was performed during the summer of 1975 on lands belonging to members of the Redwood Yield Research Cooperative. Tree selection was "semi-random". A loose criterion was adopted to select trees between 3-50 inches DBH over the range of conditions encountered on the properties sampled with the following restrictions:

1. Trees on the edges of road cuts were not measured unless the roads were recently constructed.

2. Trees were not selected in stands composed of residual old growth or in stands where there was evidence that the trees present developed as an understory of old-growth stands.
3. Trees were not selected if they were severely suppressed, had broken tops or possessed other abnormalities that would make them unlikely candidates as crop trees in managed young growth stands.

Felled tree data - Measurements from the felling of 408 redwood trees were also included as part of the volume sample. Measurements on individual trees included stump height, DBH, inside and outside bark diameter at the ends of each log, and total height in feet. This data was collected in Del Norte, Humboldt, Mendocino, and Sonoma counties by a variety of personnel between 1898 and 1965. These trees formed the sample basis used by Palley (1959, 1961) for previous redwood volume tables with the exception of 180 trees that were discarded because of suspicious or missing measurements.

Volume Computations and Standards

Dendrometry data were processed by a specially modified version of the STX computer program which produces information on tree volumes to various top diameters inside bark.^{1/} Felled tree data was also produced by STX to insure compatible volume computations.

Logs were scaled as conic frustrums for cubic volumes. Scribner volumes were based on 16 foot log lengths with a one-half foot trim allowance between logs.

Stump height for dendrometry measurements was chosen in the field to coincide where the tree would probably be cut. For felled trees, it was a constant one and a half feet.

Differences in measurement techniques and sampling distributions among dendrometered and scaled trees raised the possibility that the two redwood data sets might not be compatible. Statistical tests, however, indicated that whatever differences exist are insignificant. (See Appendix I)

Volume Equations

The general form of the volume equation used in this study is

$$V = b_0 D^{b_1} H^{b_2} \quad (1)$$

where V is tree volume in respective units, D is DBH in inches, H is total height in feet, and b_0 , b_1 , and b_2 are constants to be fitted. The constants

^{1/} Readers desiring more information on the use of optical dendrometers and the STX program are referred to the discussion and reference citations in: Wensel, Lee C. 1971. Tree Volume Equations and Tables from Dendrometer Measurements. Hilgardia, V 41, No. 4, U.C. Agric. Exp. Sta. Also see Grosenbaugh, L.R. 1967. STX-Fortran 4 program for estimates of tree populations from 3P sample tree measurements. USFS, Berk.; Res. Pap. PSW-13, revised.

Table 1

Statistical Summary of Volume Functions

$$\text{General form: } V = b_0 D^{b_1} H^{b_2}$$

	b_0	b_1	b_2	R^2	Sample Size	Average Deviation(%)	Aggregate Deviation(%)	Coeff. Variati...
Cubic foot Volumes								
Redwood	.001841	1.9678	.9887	.97	607	-.12	5.6	15.4
Douglas fir	.001697	1.6726	1.2261	.98	203	.80	4.4	11.7
Other Whitewoods	.001045	1.6759	1.3494	.99	74	-.86	4.4	11.5
Board foot Volumes								
Redwood	.001190	2.1931	1.2900	.94	584	.30	14.0	19.8
Douglas fir	.001900	1.8918	1.4220	.94	151	.41	11.1	15.8
Other Whitewoods	.000423	1.8928	1.7600	.96	38	.38	11.3	16.0

1. Coefficients estimated by weighted non-linear regression.
2. Board foot volumes: Scribner rule, 16 foot lengths, .5 feet trim allowance. Volumes are from approximate 1.5 foot stump to a 6.0 inch top inside bark.

Cubic foot Volumes: Conic frustrums, variable section lengths, no trim allowance. Volumes are from approximate 1.5 foot stump to tree tip.
3. R^2 values computed from squared deviations of unweighted data.
4. Coefficients of variation are not directly obtainable by the methods used to fit the volume equations. The values listed are approximations obtained by taking the anti-log of the square root of the residual variance of the basic volume equation fitted in logarithmic form.
5. For volumes in cubic meters, multiply b_0 in the cubic foot equations by .0283 if D and H are in English units (inches and feet, respectively) and by $(.0283)(.3937^{b_1})(3.281^{b_2})$ if D and H are in metric units (cm. and meters, respectively).

in equation (1) were fitted by weighted nonlinear regression (procedures are explained in Appendix 1). Table 1 shows pertinent statistical information and estimates of the coefficients of the volume equations for the three species groups and two volume categories. Tables 3 to 8 at the end of this report were generated by substituting various height and DBH combinations in the appropriate equations. Tables 9 to 11 indicate the number of sample trees by 2 inch DBH classes and 10 foot height classes for three species groups.

Reliability and Adjustments to Local Conditions

The volume tables essentially give "average" volumes for trees of a given DBH and height. These volumes are subject to two sources of statistical error when used for an individual tree: (1) an error associated with estimating the "true" population volume relationship from samples. This source of error can be reduced by taking more samples. (2) An error associated with random deviations about the "true" population volume relationship. This source of error may be reduced by adding additional explanatory variables (e.g., form class).

As with all regional volume tables, there is a possibility that stands of trees in specific localities may depart from the general relationships shown in these tables. Also, where scaling practices differ from the ones used in this study an additional source of bias may be introduced. Hence, field checks should be made before local use.

The following procedures are suggested to check the accuracy of volume tables at the 95 percent probability level.

1. Specify the accuracy level desired for the estimate of the mean tree volume for the stand being sampled.
2. Determine from Table 2 the number of sample trees required.

Table 2

Sample trees required for 95 percent confidence intervals of various levels of accuracy for mean tree volumes.

Accuracy (%)	Cubic Feet	Board Feet
10	9	16
8	14	25
6	25	44
4	56	100
2	225	400
Assumed Coefficient of Variation	15	20

3. Once the sample size is chosen, trees should be selected for volume measurement that are representative of the trees for which volume estimates are desired. Each sample tree is measured for DBH, total height and volume. Volume can be obtained with Relaskops, dendrometers, or by scaling felled trees. Volumes of sample trees should be determined by the same procedures used in scaling operations.
4. Compute the percent difference between the measured volumes and those estimated by the volume tables for each sample tree. Then compute the average percent difference for all sample trees.
5. If the absolute value of the average percent difference, $|\bar{P}|$, is less than the accuracy level, use the tables without adjustment.
6. If the $|\bar{P}|$ is greater than the accuracy level and there are no trends in the percent deviations, multiply estimated volumes by the factor

$$F = (1 + \bar{P}/100)$$

Percent deviations sometimes show trends with estimated volume or DBH in which case regression adjustments may be utilized. For example, if the percentage volume deviation of the sample varies with DBH in a linear fashion, the following correction factor may be used

$$F_{D_i} = [1 + (\bar{P} + b(D_i - \bar{D}))]/100$$

where

$$F_{D_i} = \text{correction factor for a tree of } D_i \text{ inches}$$

$$\bar{D} = \text{average DBH of volume sample trees}$$

b = regression constant computed from the volume sample

$$= \frac{\sum (P_j - \bar{P})(D_j - \bar{D})}{\sum (D_j - \bar{D})^2}$$

P_j = Percent deviation of the j^{th} tree in the volume sample

D_j = DBH of the j^{th} tree in the volume sample

If more refined estimates or different confidence levels are desired, the coefficients of variation listed in Table I for each species-volume group can be utilized in conjunction with the familiar sample size formula.

7. As an example, suppose 8 percent accuracy levels were desired at the 95 percent probability level. A redwood cubic volume check yielded the following data:

Sample Tree No.	DBH	Total Height	Volumes from Tables	Volumes Scaled	Percent Difference
1	12	60	14	14.8	+5.7
2	12	70	16	17.1	+6.8
3	12	90	21	22.8	+4.8
4	18	90	46	49.4	+7.4
5	20	130	82	88.3	+7.5
6	22	110	84	91.7	+9.2
7	28	100	123	134.2	+9.1
8	28	130	160	175.5	+9.7
9	30	120	169	185.9	10.0
10	34	130	234	260.4	11.3
11	36	130	262	290.8	11.0
12	36	140	282	314.4	11.5
13	44	160	477	538	12.8
14	44	160	477	540.9	13.4
Averages	26.8				+9.3

In this example 9.3 is greater than the 8 percent accuracy requirement and may be used to compute the correction factor

$$F = 1 + 9.3/100 = 1.093$$

There is also a trend for percent differences to increase with DBH. In this case, a regression correction factor that varies with DBH will be more precise. The regression coefficient "b" for the correction factor F_{D_i} is computed as

$$b = \frac{[(5.7-9.3)(12-28.6)+(6.8-9.3)(12-28.6)+\dots+(13.4-9.6)(44-28.6)]}{[(12-28.6)^2 + (12-28.6)^2 + \dots + (44-28.6)^2]}$$

$$= .227$$

The correction factor is then

$$F_{D_i} = [1+(9.3+.227(D_i-26.8))]/100]$$

$$= 1.093 + .00227(D_i-26.8)$$

For a 38 inch tree, the volume estimate from the table would be adjusted upward by the factor

$$F_{38} = 1.093 + .00227(38-26.8)$$

$$= 1.118$$

Literature Cited

Palley, M. N.

- 1959 Board-foot volume tables for young growth coastal redwood. Calif. For. and Forest Products 11:1-6.
- 1961 Cubic-foot volume tables for young growth coastal redwood. Calif. For. and Forest Products 28:1-6.

Table 3 REDWOOD
GROSS CUBIC FOOT VOLUME TABLE
VOLUME FROM 1.5' STUMP TO TIP

DBH (INCHES)	TOTAL HEIGHT (FEET)																	
	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
4	1	1	1	2	2	2	2	3	3	3	3	4	4	4	5	5	5	5
6	2	2	3	4	4	5	5	6	7	7	8	8	9	9	10	11	11	12
8	3	4	5	6	7	8	9	10	11	13	14	15	16	17	18	19	20	21
10	5	7	8	10	11	13	15	16	18	19	21	23	24	26	27	29	31	32
12	7	9	12	14	16	19	21	23	26	28	30	32	35	37	39	42	44	46
14	10	13	16	19	22	25	28	31	35	38	41	44	47	50	53	56	59	62
16	12	17	21	25	29	33	37	41	45	49	53	57	61	65	69	73	77	81
18	16	21	26	31	36	41	46	52	57	62	67	72	77	82	87	92	97	102
20	19	26	32	38	45	51	57	63	70	76	82	89	95	101	107	114	120	126
22	23	31	39	46	54	61	69	77	84	92	99	107	114	122	129	137	144	152
24	28	37	46	55	64	73	82	91	100	109	118	127	136	145	154	163	171	180
26	32	43	54	64	75	85	96	106	117	127	138	148	159	169	180	190	201	211
28	37	50	62	74	87	99	111	123	135	147	160	172	184	196	208	220	232	244
30	43	57	71	85	99	113	127	141	155	169	183	197	211	224	238	252	266	280
32	49	65	81	97	113	128	144	160	176	192	207	223	239	255	271	286	302	318
34	55	73	91	109	127	145	163	180	198	216	234	252	269	287	305	323	340	358
36	61	82	102	122	142	162	182	202	222	242	262	282	301	321	341	361	381	401
38	68	91	113	135	158	180	202	224	247	269	291	313	335	357	379	401	423	445
40	76	100	125	150	175	199	224	248	273	297	322	346	371	395	420	444	468	493
42	83	110	138	165	192	219	246	273	300	327	354	381	408	435	462	489	516	542
44	91	121	151	181	211	240	270	300	329	359	388	418	447	477	506	536	565	594
46	99	132	165	197	230	262	295	327	359	392	424	456	488	520	553	585	617	649
48	108	144	179	215	250	285	320	356	391	426	461	496	531	566	601	636	671	705
50	117	156	194	232	271	309	347	385	423	461	499	537	575	613	651	689	727	765

Table 4 REDWOOD
GROSS BOARD FOOT VOLUME TABLE
VOLUME FROM 1.5' STUMP TO 6" TOP (INSIDE BARK)

DBH (INCHES)	TOTAL HEIGHT (FEET)																	
	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
10	15	22	29	36	45	53	62	71	80	89	99	109	119	129	140	151	161	172
12	22	32	43	54	66	79	92	105	119	133	148	162	178	193	209	225	241	257
14	31	45	60	76	93	111	129	149	167	187	207	228	249	271	293	315	338	361
16	42	61	81	102	125	148	173	198	224	250	277	305	334	363	392	422	453	484
18	54	79	105	132	162	192	223	256	290	324	359	395	432	469	508	546	586	626
20	68	99	132	167	204	242	282	323	365	408	452	498	544	591	640	689	738	789
22	84	122	163	206	251	298	347	398	450	503	558	614	671	729	788	849	910	972
24	102	148	197	249	304	361	420	481	544	609	675	743	812	882	954	1027	1101	1177
26	121	176	235	297	362	430	501	573	649	726	804	885	968	1052	1137	1224	1313	1402
28	143	207	276	349	426	506	589	675	763	854	946	1041	1138	1237	1338	1440	1544	1650
30	166	241	321	406	495	589	685	785	888	993	1101	1211	1324	1439	1556	1675	1796	1919
32	191	277	370	468	571	678	789	904	1023	1144	1268	1396	1526	1658	1793	1930	2070	2211
34	219	317	422	534	652	774	902	1033	1168	1307	1449	1594	1743	1894	2048	2205	2364	2526
36	248	359	479	606	739	878	1022	1171	1324	1481	1642	1807	1975	2147	2321	2499	2679	2863
38	279	404	539	687	837	988	1151	1318	1491	1668	1849	2035	2224	2417	2614	2814	3017	3223
40	312	452	603	763	931	1106	1288	1475	1668	1866	2069	2277	2489	2705	2925	3149	3376	3607
42	347	503	671	849	1036	1231	1433	1642	1856	2077	2303	2534	2770	3010	3255	3504	3757	4014
44	385	558	743	941	1148	1363	1587	1818	2056	2300	2550	2806	3067	3334	3605	3881	4161	4445
46	424	615	820	1037	1265	1503	1749	2004	2266	2536	2811	3093	3381	3675	3974	4278	4587	4901
48	466	675	900	1138	1389	1650	1921	2200	2488	2784	3086	3396	3712	4034	4363	4696	5036	5384
50	509	738	984	1245	1519	1804	2100	2406	2721	3044	3375	3714	4060	4412	4771	5136	5507	5884

Table 5 DOUGLAS FIR
GROSS CUBIC FOOT VOLUME TABLE
VOLUME FROM 1.5' STUMP TO TIP

DBH (INCHES)	TOTAL HEIGHT (FEET)																	
	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
4	1	2	2	3	3	4	4	5	5	6	7	7	8	9	9	10	11	11
6	2	3	4	5	6	7	8	10	11	12	13	15	16	17	18	20	21	23
8	4	5	7	8	10	12	14	16	18	19	21	24	25	28	30	32	34	36
10	5	7	10	12	15	17	20	23	25	28	31	34	37	40	43	47	50	53
12	7	10	13	16	20	23	27	31	34	38	42	46	50	55	59	63	67	72
14	9	13	17	21	26	30	35	40	45	50	55	60	65	71	76	82	87	93
16	11	16	21	27	32	38	44	50	56	62	68	75	82	88	95	102	109	116
18	14	20	26	32	39	46	53	60	68	76	83	91	99	108	116	124	133	141
20	16	23	31	39	47	55	63	72	81	90	99	109	119	128	138	148	158	169
22	19	28	36	45	55	64	74	85	95	106	117	128	139	151	162	174	186	198
24	22	32	42	52	63	74	86	98	110	122	135	148	161	174	188	201	215	229
26	26	36	48	60	72	85	98	112	126	140	154	169	184	199	214	230	246	262
28	29	41	54	68	82	96	111	127	142	158	175	191	208	225	243	260	278	296
30	32	46	61	76	92	108	125	142	160	178	196	215	234	253	272	292	312	332
32	36	51	68	85	102	120	139	158	178	198	218	239	260	282	303	325	348	370
34	40	57	75	94	113	133	154	175	197	219	242	265	288	312	336	360	385	410
36	44	63	82	103	124	147	169	193	217	241	266	291	317	343	369	396	423	451
38	48	69	90	113	136	160	185	211	237	264	291	319	347	375	404	434	464	494
40	53	75	98	123	148	175	202	230	259	287	317	347	378	409	441	473	505	538
42	57	81	107	133	161	190	219	249	280	312	344	377	410	444	478	513	548	584
44	62	88	115	144	174	205	237	270	303	337	372	407	443	480	517	554	592	631
46	66	94	124	155	188	221	255	290	326	363	401	439	477	517	557	597	638	679
48	71	101	133	167	201	237	274	312	351	390	430	471	513	555	598	641	685	730
50	76	109	143	178	216	254	293	334	375	418	461	504	549	594	640	686	734	781

Table 6 DOUGLAS FIR
 GROSS BOARD FOOT VOLUME TABLE
 VOLUME FROM 1.5" STUMP TO 6" TOP (INSIDE BARK)

DBH (INCHES)	TOTAL HEIGHT (FEET)																	
	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
10	19	20	39	50	62	75	89	103	118	134	150	167	184	202	220	239	258	277
12	26	40	54	71	88	106	126	146	167	189	212	236	260	285	311	337	364	391
14	35	53	73	95	118	142	168	195	224	253	284	315	348	381	416	451	487	524
16	45	68	94	122	152	183	217	252	288	326	365	406	448	491	535	581	627	674
18	57	85	117	152	189	229	271	314	360	408	457	507	560	614	669	725	783	843
20	69	104	143	186	231	279	330	384	440	497	557	619	683	749	816	885	956	1029
22	83	125	172	222	277	335	396	460	526	596	668	742	818	897	978	1060	1145	1232
24	98	147	202	262	326	395	467	542	621	702	787	874	965	1057	1152	1250	1350	1452
26	114	171	235	305	380	459	543	630	722	817	916	1017	1122	1230	1341	1454	1571	1690
28	131	197	271	351	437	528	624	725	831	940	1053	1171	1291	1415	1543	1673	1807	1944
30	149	225	308	400	498	602	712	827	946	1071	1200	1334	1471	1613	1758	1907	2059	2215
32	169	254	348	452	562	680	804	934	1069	1210	1356	1507	1662	1822	1986	2154	2326	2502
34	189	285	391	507	631	763	902	1047	1199	1357	1521	1690	1864	2043	2227	2416	2609	2807
36	211	317	435	564	703	850	1005	1167	1336	1512	1695	1883	2077	2277	2482	2692	2907	3127
38	233	351	482	625	778	941	1113	1293	1480	1675	1877	2086	2301	2522	2749	2982	3220	3464
40	257	387	532	689	858	1037	1226	1424	1631	1846	2068	2296	2535	2779	3029	3286	3548	3817
42	282	424	583	755	941	1137	1345	1562	1789	2024	2268	2521	2780	3048	3322	3603	3891	4186
44	308	463	637	825	1027	1242	1468	1706	1953	2211	2477	2752	3036	3328	3628	3935	4249	4571
46	335	504	692	897	1117	1351	1597	1855	2125	2405	2694	2994	3303	3620	3946	4280	4622	4972
48	363	546	750	973	1211	1464	1731	2011	2303	2606	2920	3245	3579	3923	4277	4639	5010	5389
50	392	590	811	1051	1308	1582	1870	2172	2488	2815	3155	3505	3867	4238	4620	5011	5412	5821

Table 7 WHITEWOODS
 GROSS CUBIC FOOT VOLUME TABLE
 VOLUME FROM 1.5' STUMP TO TIP

DBH (INCHES)	TOTAL HEIGHT (FEET)																	
	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
4	1	2	2	3	3	4	5	5	6	7	8	8	9	10	11	12	13	14
6	2	3	4	5	6	8	9	11	12	13	15	17	18	20	22	23	25	27
8	3	5	7	9	11	13	15	17	19	22	24	27	29	32	35	38	41	43
10	5	7	10	12	15	18	21	25	28	32	35	39	43	47	51	55	59	63
12	7	10	13	17	21	25	29	34	38	43	48	53	58	63	69	74	80	86
14	9	13	17	22	27	32	38	44	49	56	62	69	75	82	89	96	103	111
16	11	16	21	27	34	40	47	54	62	70	78	86	94	103	111	120	129	139
18	13	19	26	33	41	49	58	66	75	85	94	104	115	125	136	147	159	169
20	16	23	31	40	49	59	69	79	90	101	113	125	137	149	162	175	189	202
22	18	27	36	47	57	69	81	93	106	119	132	146	160	175	190	205	221	236
24	21	31	42	54	66	79	93	107	122	137	153	169	186	202	220	237	255	274
26	24	36	48	62	76	91	107	123	140	157	175	193	212	232	251	271	292	313
28	27	40	55	70	86	103	121	139	158	178	198	219	240	262	284	307	331	354
30	31	45	61	78	96	115	135	156	177	200	222	246	270	294	319	345	371	398
32	34	51	68	87	107	129	151	174	198	222	248	274	301	328	356	384	413	443
34	38	56	76	97	119	142	167	192	219	246	274	303	333	363	394	425	458	490
36	42	62	83	106	131	157	184	212	241	271	302	334	366	399	433	468	504	540
38	46	67	91	116	143	172	201	232	264	297	330	365	401	437	475	513	551	591
40	50	73	99	127	156	187	219	253	287	323	360	398	437	477	517	559	601	644
42	54	80	108	138	170	203	238	274	312	351	391	432	474	517	561	606	652	699
44	58	86	116	149	183	219	257	297	337	379	422	467	512	559	607	655	705	756
46	63	93	125	160	197	236	277	319	363	409	455	503	552	602	654	706	760	814
48	68	100	135	172	212	254	298	343	390	439	489	540	593	647	702	758	816	874
50	72	107	144	184	227	272	319	367	418	470	523	578	635	693	752	812	873	936

Table 8. WHITEWOODS
 GROSS BOARD FOOT VOLUME TABLE
 VOLUME FROM 1.5' STUMP TO 6" TOP (INSIDE BARK)

DBH (INCHES)	TOTAL HEIGHT (FEET)																	
	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
10	13	22	32	45	58	74	91	110	130	151	174	198	224	250	279	308	339	371
12	19	31	46	63	83	104	128	155	183	213	245	280	316	354	394	435	479	524
14	25	41	61	84	111	140	172	207	245	285	329	374	423	474	527	583	641	701
16	32	53	79	108	142	180	221	267	315	367	423	482	544	610	678	750	825	903
18	40	66	98	136	178	225	277	333	394	459	529	602	680	762	848	937	1031	1128
20	49	81	120	166	217	275	338	407	481	561	645	735	830	930	1035	1144	1259	1377
22	59	97	144	198	260	329	405	487	576	671	773	881	994	1114	1239	1371	1507	1650
24	69	114	170	234	307	388	477	574	679	792	911	1038	1172	1313	1461	1616	1777	1945
26	80	133	197	272	357	451	555	668	790	921	1060	1208	1364	1528	1700	1880	2068	2263
28	92	153	227	313	410	519	639	769	909	1060	1220	1390	1570	1758	1956	2163	2379	2604
30	105	175	259	357	466	592	728	876	1036	1208	1390	1584	1789	2004	2229	2465	2711	2967
32	119	197	292	403	520	668	822	990	1171	1365	1571	1790	2021	2264	2519	2786	3064	3353
34	133	221	328	452	593	750	922	1110	1313	1530	1762	2007	2267	2539	2825	3124	3436	3761
36	149	247	365	503	660	835	1028	1237	1463	1705	1963	2237	2526	2829	3148	3481	3829	4190
38	165	273	405	558	732	925	1139	1371	1621	1889	2175	2478	2798	3134	3487	3856	4241	4642
40	181	301	446	615	806	1020	1255	1510	1786	2082	2397	2730	3083	3454	3843	4249	4674	5115
42	199	330	489	674	884	1118	1376	1656	1959	2283	2628	2995	3381	3788	4215	4661	5126	5610
44	217	361	534	736	966	1221	1503	1809	2139	2493	2878	3278	3693	4137	4602	5088	5598	6127
46	236	392	581	801	1050	1329	1635	1968	2327	2712	3122	3557	4017	4500	5006	5536	6089	6664
48	256	425	630	868	1138	1440	1772	2133	2522	2940	3384	3856	4354	4877	5426	6001	6600	7223
50	277	459	680	938	1230	1556	1914	2304	2725	3176	3656	4166	4703	5269	5862	6483	7130	7804

Table 9 NUMBER OF SAMPLE TREES
GROSS CUBIC-FOOT VOLUME TABLE
REDWOOD

DBH (INCHES)	TOTAL HEIGHT (FEET)																				TOTAL			
	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210		220	230	
	(NUMBER OF TREES)																							
4	1	1																						2
6	4																							4
8		3	6																					9
10			1	2	1																			4
12			1	2		3																		6
14				1	4	15	7	5	3	3													1	39
16				1	3	11	9	7	2	5	2													48
18						4	6	6	10	4	9	2												41
20				1	1	6	6	10	4	6	9	3	1											47
22						1	2	2	7	9	6	10	7	1										45
24							1		12	10	13	9	4	1	1									51
26						1		4	7	11	10	12	7	2	1	1								56
28							1	1	6	15	16	8	10	2	1		2							61
30									1	5	13	16	6	5	4	3	1	1						55
32								1	4	3	13	12	2	6	5		2							53
34									2	8	14	12	4	2	1	4								47
36								1		1	5	7	3	2	3	2			2					26
38																			1		1			2
40												1					2		1					4
42																3		2		1			1	7
44															1	1								3
46												1				1								2
48															1									1
50											1													1
52																1								1
TOTAL	5	4	18	7	9	41	32	37	58	85	111	93	44	21	17	16	7	7	4	0	1		1	687

Table 10 NUMBER OF SAMPLE TREES
GROSS CUBIC-FOOT VOLUME TABLE
DOUGLAS FIR

DBH (INCHES)	TOTAL HEIGHT (FEET)																				TOTAL		
	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210		220	
	(NUMBER OF TREES)																						
4	8	1	1																				22
6		9	2																				14
8		6	2	1	1																		10
10			1	2	1																		4
12			1	1	1			1	1														5
14							2	3	1														6
16							1	1	2	3	1												11
18							1	1	2	4	3	2											16
20									2	1	4	6	5	1			1						20
22								1	2	5	6	1	2										17
24									1	5	8	4	1										19
26								1	5	5	2	5	1	2	1								26
28											2	4	1	2	1				1				19
30											1	2	1	1				1					8
32													2	1		1	2	2	1				8
34													2		1	1					1		5
36													1			1	1	1	2				6
38																	1						1
40																	1						2
42																							0
44																							0
46																							0
48																							0
50																						1	1
TOTAL	8	16	16	6	2	3	8	2	6	11	11	27	29	28	7	4	8	5	5	2	1		203

Table 11 NUMBER OF SAMPLE TREES
GROSS CUBIC-FOOT VOLUME TABLE
OTHER WHITEWOODS

DBH (INCHES)	TOTAL HEIGHT (FEET)																		TOTAL
	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	
	(NUMBER OF TREES)																		
4	6	8																	14
6		6	3	1		1													11
8		1		3	1														5
10			1	1	2														4
12			1	1		2			2										6
14							1	1											2
16					2					1									3
18							1		1	1									3
20									1		1		1						4
22										1			1	1					4
24										1						1			2
26											1	1				1			3
28											1	2	1	1	2				7
30											1		1	1					4
32																	2		2
34													1						1
36																		1	1
38																			0
40														1					1
TOTAL	6	15	5	6	5	3	2	0	4	2	7	3	5	4	2	2	2	1	74

Appendix A. Statistical Procedures

The coefficients of the volume equations used in preparing this report were estimated by weighted non-linear regression. The computer routine employed was ZXSSQ of the International Mathematics and Statistical Laboratories, available through the CDC-6400 operating system of the University of California in Berkeley. The procedure is a modified Levenberg-Marquardt algorithm.

A. Derivation of Weights

It is well known that the variance of tree volume increases with DBH and height. Consequently, to stabilize the error variance in regression analysis, each observation is weighted by a factor inversely proportional to its expected variance.

Initially, for each of the six volume equations, the sample trees were segregated by two inch diameter classes and ten foot height classes. For each of these groups, the volume variance, average DBH, and average total height were computed. Subsequently, the following variance function was fitted after weighting each observation by the degrees of freedom of each group variance.^{1/}

$$\log(S_{v_i}^2) = a_0 + a_1 \log \bar{D}_i + a_2 \bar{H}_i \quad (2)$$

where:

\log signifies the "natural" logarithm

$\log(S_{v_i}^2)$ = the log of the variance of volume of group "i"

\bar{D}_i = average DBH of group "i"

\bar{H}_i = average height of group "i"

a_0, a_1, a_2 = fitted constants

Table A shows the estimated coefficients of the variance function for each of the six volume equations.

^{1/} See The Construction of Standard Tree Volume Tables by Weighted Multiple Regression, by Douglas J. Gerrard, Faculty of Forestry Technical Report No. 6, University of Toronto, 1966.

Table A

Coefficients of the Volume Variance Function

	Cubic Volumes				Board foot Volumes			
	a ₀	a ₁	a ₂	R ²	a ₀	a ₁	a ₂	R ²
Redwood	-1.96	.139	.020	.89	-.071	1.86	.364	.92
Douglas fir	-3.63	.149	.025	.69	.363	.148	.038	.89
Other whitewoods	-7.66	.367	.013	.65	.573	.490	-.035	.75

Given a sample tree of DBH equal to D_j and a total height of H_j , the weight assigned to this tree was computed as

$$w_j = [e^{(a_0 + a_1 D_j + a_2 H_j)}]^{-1}$$

where e is the base of the natural logarithms (2.7183).

B. Bark Taper Assumptions

Dendrometers can only be used to measure outside bark diameters of upper stems. These values must be adjusted to obtain inside bark scaling diameters for volume determination. Adjustments were made by computing the bark factor at breast height for each tree.

$$BF = \text{DBH outside bark} / \text{DBH inside bark}$$

This factor was assumed to be the same at all points on the tree stem.

To test the validity of this assumption, 108 redwood trees were selected at random from the felled tree sample. For each tree, cubic volumes based on the bark factor assumption were computed (VBF) and paired with volumes based on actual inside bark scaling diameters (VA). The following function was fitted

$$VA = C_0 + C_1 VBF$$

As expected, the constant term was insignificant but the slope term (C_1) was significantly different from 1 and indicated that the bark factor assumption resulted in a three percent underestimate.

Brickell^{2/} and others, however, have noticed that dendrometer measurements of stem diameters give consistent overestimates when compared to caliper measurements of the same points. No reason for this discrepancy is evident but it is compounded into an overestimate of cubic volumes of somewhere between two and five percent when compared to scaled tree measurements.

Hence, for the redwood trees in this study, these two sources of bias may be compensatory. Further analysis of covariance, comparing the differences in residual variances from the volume regression using both dendrometry and scaled tree data with the pooled variance derived from fitting separate regressions to each data set, also indicated that statistical differences between the two data sets were insignificant at critical levels of .15 or less.

Hence, sample volumes for redwood dendrometry trees were based on a constant bark factor determined at breast height and were combined with scaled tree data in developing final volume equations.

^{2/} Brickell, J.E. 1976. Bias and precision of the Barr and Stroud dendrometer under field conditions. USDA For. Ser. Res. Paper INT-186, 46 p., 83 ref.