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A TREE INCREMENT MODEL SYSTEM FOR NORTH COASTAL CALIFORNIA

DESIGN AND IMPLEMENTATION

by

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Abstract

Model form and estimated coefficients are presented for predicting five year diameter, height, and crown base increment for some major species groups found in the North Coast Region of California. Secondary "modifier" models are also documented. In addition, crown base estimators are presented for use in situations where this tree characteristic was not measured.

Coupled with models presented in previous research notes, the models and procedures described here constitute a complete equation system for a distance-independent tree-based growth and yield model.

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I. INTRODUCTION

This research note provides a description and derivation of the current state[1] of a model system for predicting individual tree height, basal area, and crown size increment of young growth redwood and Douglas fir in the North Coast region of California. In addition, crown size estimators are also described for use in situations where crown measurements are unavailable.

These models constitute a major portion of the primary system of equations used to drive CRYPTOS type computer programs (see Res. Note No. 14)[2]. In the current vernacular of forest growth modelling, this system can be classified as a distance independent tree growth model. Individual trees are the basic unit of forest growth analysis and spatial distributions of trees are not explicitly recognized. Other models needed to complete this system are mortality predictors (Res. Note No. 6), site index equations (Res. Note No. 5), and tree volume equations (Res. Note No. 9).

This system has been designed to be independent of tree or stand ages and can be used to model the growth of even or uneven-aged stands.

Figure 1. provides a conceptual schematic of how the models presented in this report are linked within the CRYPTOS program.

These models represent a fourth generation attempt at developing this type of a system. These models have been extensively tested and are considered to give reasonable growth predictions, at least for stand types from which the basic data was drawn. After some experience has been gained with these models in operational situations, they may be revised by potential users to reflect 'local' or alternative conditions.

II. SPECIES GROUPS

At an early stage in this study, eight species groups were recognized for modelling purposes. The choice of species groupings is essentially a compromise between a) relative abundance and commercial importance of individual species; b) the availability of data for modelling purposes; and c) relative similarities and differences in terms of growth characteristics. These species groups are listed as follows:

1) young growth redwood
2) young growth Douglas fir

1. As with any modelling effort of this type, the model system is under a state of constant change and modification in light of new and better data, experience gained through use of the system, and subsequent evolution in the system design.
2. Research Notes are listed by number in the Literature Cited.

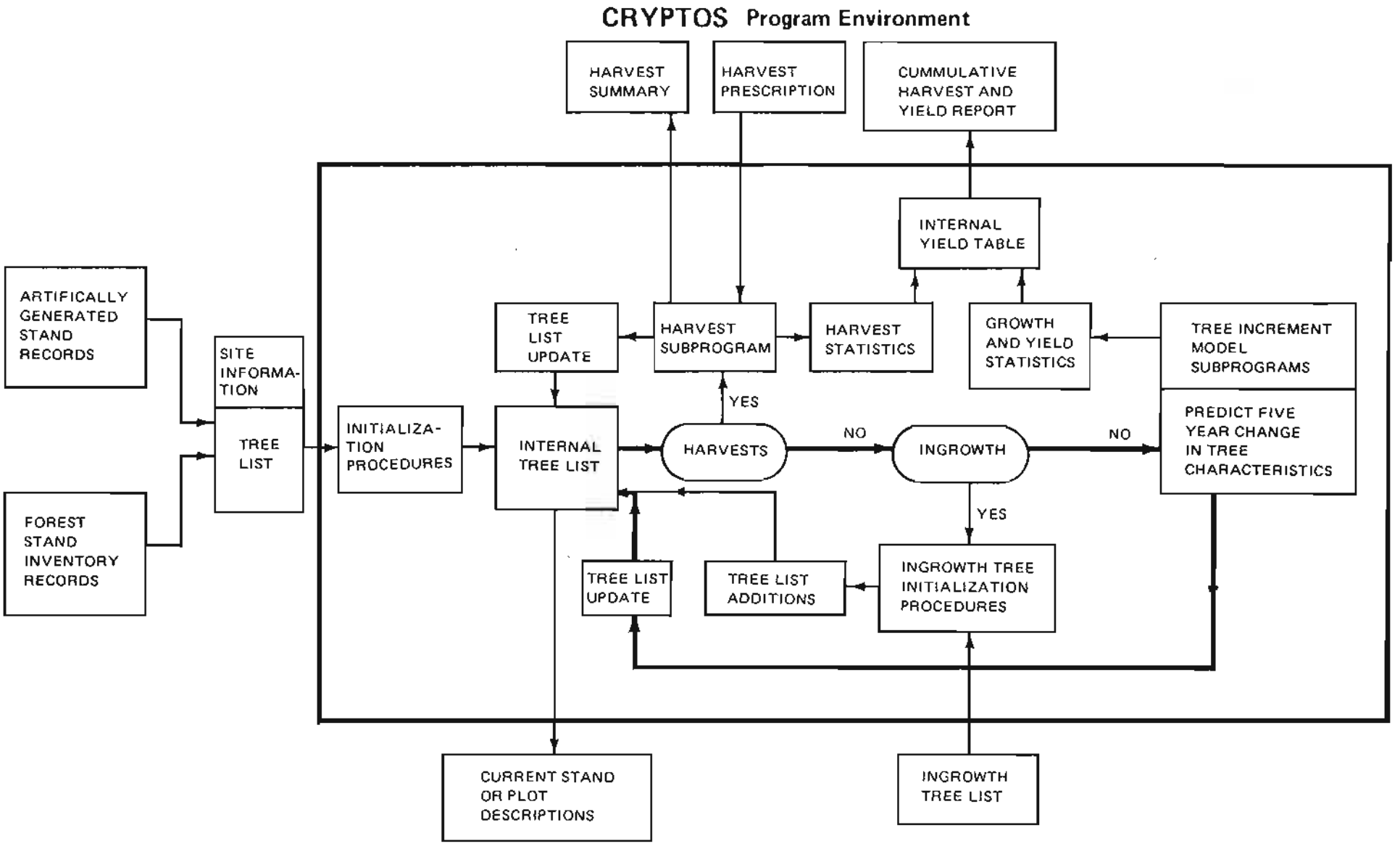

Figure 1. CONCEPTUAL FLOW CHART INDICATING GROWTH MODEL LINKAGES WITHIN THE CRYPTOS PROGRAM

3) other young growth conifers (mostly grand fir but includes Sitka Spruce and Western Hemlock
4) Tan oak
5) Red alder
6) other hardwoods (mostly madrone)
7) old growth Redwood
8) other old growth conifers (mostly Douglas fir)

Based on a consensus of the Redwood Coop advisory panel, a somewhat arbitrary decison was made relative to growth in old growth species groups: old growth conifers are presummed not to grow, die, or in any way change dimensions within single five year growth periods. This decision was based on several factors:

1) There is a paucity of growth data for old growth species.

2) The growth on these trees is slow relative to young growth and it was felt that developmental efforts should be concentrated at this stage on young growth.

3) Most cooperators have indicated that most of their old growth reserves will probably be liquidated in the next twenty years so the "zero growth" assumption will not have much of an impact.

Hence, old growth species are considered part of the standing inventory and contribute to competition of young growth species but have zero growth and mortality

At this time, we have devoted most of our effort to the development of increment equations for young growth redwood and Douglas fir. Some empirical evidence suggests that the growth of other conifers in the region, especially grand fir (Abies grandis), is very similar to Douglas fir in growth habits. Coupled with the general lack of growth data for the minor conifers, our provisonal plans are to assume that they grow like Douglas fir.

Hardwood species however are noticably different. Crown size estimators have been developed for tanoak (Lithocarpus densiflora) and are described later in this report. Growth models for tanoak however are still being developed. The current lack of adequate data for alder (Alnus rubra) has necessitated abandoning direct development of growth models for this species. We may eventually develop some ad hoc estimators later for the sake of completeness. The "other hardwood" species group will eventually be treated as tanoak in terms of growth and yield.

III. DATA SOURCES

The data used to derive the model coefficients presented in this report has been drawn from an extensive record collection of permanent and temporary growth plots located in Mendocino, Humboldt, and Del Norte counties. All of these plots were within the redwood-Douglas fir forest type and situated in stands of predominantly young-growth timber.

Approximately 15% of the sample plots had one or more residual old-growth tree. About 70% of the plots were from the coastal zone that is subject to fog influence and the remainder from the somewhat drier interior. Plots that were in the transition zone to the mixed conifer forest type were excluded from analysis. With the exception of Jackson State Forest in Mendocino County, all plots were located on private forest land.

Plots were all fixed area plots ranging in size from one tenth to one half acre. For approximately two thirds of the plots, subplots were included for the measurement of smaller trees (less than 11.0 inches DBH).

Altogether, 512 plots were considered to be usable in one form or another for model development. Approximately 25% of the available plots had been partially harvested prior to measurement. These plots were screened from a much larger set with rejections predominantly based on the following items:

a) Data collection procedures were too extensive to give adequate measurements on individual trees.

b) Collection procedures on individual plots were incompatible between measurements.

c) Plots were not located in stand conditions generally representative of the coastal forest type or otherwise were felt to be of limited analytical use (highways or landings were located within plot boundaries; plots had been purposely located in unusually exceptional stand locations; plots were located in swamps, between cover type boundaries, or in situations where the treatment history was not uniform throughout the plot; plots were located in stands with exceptional amounts of wind throw, animal or logging damage, landslides had occurred within the plots, or in general, the plot was not representative of a stand condition foresters would consider managing (pygmy forest land for example)).

d) Minimum DBH's recorded on the plots were not considered to be low enough to adequately represent the within plot stocking.

In general, not all plots were used equally in developing the models presented in this report because of missing measurements or otherwise did not provide the necessary measurements for analysis. The general procedure followed in selecting trees from plots for modelling was as follows:

a) Flag trees of a given species on plots that had all of the measurements required for the model in question

b) Before further consideration, a check was made to insure the flagged subset provided a representative cross section of the species on the plot

c) Randomly draw a representative sample by species from this subset for further analysis.

A further description of data adjustments and sample selection is detailed in Appendix I.

IV. RATIONALE AND ARCHITECTURE OF THE MODEL SYSTEM

A. Data Requirements

The increment equations described in this report have been derived from growth plot data and consequently, their primary purpose is in modelling the increment of individual trees on plots or in some situations, a "modified" stand table derived from several plots in a particular stand. Application limitations and plot and tree information which is required to provide input for these models is described in Research Note No. 14. Briefly, this information is comprised of the following items:

1. Plot Information

Fifty year (breast high) site indices for the following species groups: redwood, Douglas fir, alder, and tanoak. Adequate functioning of the CRYPTOS programs requires all four site indices even though a particular species may not be present. Utilizing site conversion equations found in Appendix II of Research Note No. 11, the minimal amount of site information required to insure proper functioning of the models is either redwood or Douglas fir site index (see Research Note No. 5).

2. Tree Information

The tree information needed for model input consists of the following items for each measured tree:

1) Species code
2) DBH to nearest tenth of an inch
3) Total height to nearest foot
4) Live crown ratio
5) Tree weight on a per acre basis

Foresters are generally familiar with procedures utilized to convert plot tree measurements into total plot volumes or volumes by log sizes. The purpose of the increment equations and associated computer models is to provide some estimate of a tree by tree plot inventory record if the plot had been remeasured at some time in the future. Differences between successive plot inventory estimates provides an estimate of net plot growth.

The increment equations described in this report are for a time span of five years. Predictions for multiples of five years are accomplished by a recursive process of repeated application of the increment models.

B. Model System Architecture

The tree growth model system is composed of four main increment expressions for each species. These equations are used to estimate changes in tree DBH, total height, crown ratio, and per acre weights. Mortality enters the system by changing the per acre weights associated with each tree. As implicit expressions, these equations can be represented as:

$$CDS5_{ijkl} = f_d\{xd_{ijkl}, \theta d_k\}\{Md_{ijk}\}$$

$$HG5_{ijkl} = f_h\{xh_{ijkl}, \theta h_k\}\{Mh_{ijk}\}$$

$$CBG5_{ijkl} = f_c\{xc_{ijkl}, \theta c_k\}$$

$$PD_{ijkl} = f_p\{xp_{ijkl}, \theta p_k\}$$

where

ijkl	indices denoting the jth tree on the ith plot of the kth species during the lth five year growth period. Subsequent indexing of expressions is deleted for the sake of conciseness.
CDS5	Five year change in tree DBH squared, outside bark, in square inches.
HG5	Five year change in total height in feet.
CBG5	Five year change in height to the base of the live crown in feet[1].
PD	Probability of the tree will die during the next year[2].

1. In field determination of crown lengths, trees with asymmetrical crowns or "holes" in the foliage were visually reapportioned up the stem to get an approximation of an average complete crown length
2. Tree mortality models were described in Research Note No.6.

f_d, f_h, f_c, f_p Denote corresponding implicit functions and will subsequently be referred to as structural components.

xd,xh,xc,xp Denote vectors of explanatory variables used in estimating the appropriate increment variable.

$\theta d, \theta h, \theta c, \theta p$ Denote species specific vectors of structural parameters that are estimated from the data and determine the level of the increment estimate.

Md,Mh Denote equation modifiers that are used to alter the predictions. An explanation of these equation modifiers will be given shortly.

C. Estimates of the Future Tree List

Given the set of models previously described, each tree represented in the tree list has its characteristics updated by the following conventions to get an estimate of what it would look like if it was remeasured five years later.

Future tree DBH. If D_1 is the current tree DBH, an estimate of its DBH five years later (D_2) is given by

$$D_2 = \{D_1^2 + CDS5\}^{1/2}$$

Future total height. If HT_1 is the current total height, height five years later (HT_2) is estimated as

$$HT_2 = HT_1 + HG5$$

Future crown ratios. If CR_1 is the current crown ratio, the crown ratio after five years (CR_2) is estimated by

$$CR_2 = \{(CR_1)(HT_1) - CBG5 + HG5\}/HT_2$$

Future per acre weights. If the current per acre weight is W_1, the weight five years later is estimated as

$$W_2 = W_1\{1. - 5.(PD)\}$$

D. Development of Structural Components

In developing the structural components of our increment equations, particularly for diameter and height increment, we have taken the point of view that the system of models being developed represents the interactions of trees with their environment (mainly other trees). By design and intended use of this system, we implicitly attach a causal interpretation to the models: a change in a tree's environment or characteristics through a simulated harvest or the normal course of tree

and stand development causes some form of tree growth response. Consequently, we have designed these components to be biologically interpretable and have relied mainly on crown size and crown ratios as dominant explanatory variables as these characteristics are associated with photosynthate producing capabilities of trees. Secondly, the major use of this system of models is for stand conditions which currently do not exist (e.g. stands that might like to have but are currently unavailable, or predictions far into the future for currently existing stands). Plausible predictions in these situations requires explicit consideration of underlying biological processes.

The structural components of these increment models can be thought of as having two parts: a) a tree potential and b) a reduction for competition. Functionally, the tree potential represents the maximum growth a tree could obtain given its current characteristics in an open grown environment. As the proximity of other trees induces some form of growth reduction, the competition component scales the potential growth in a multiplicative fashion from relative weight of '1' in open grown conditions towards '0' as the tree begins to become overtopped in dense stand conditions. Our approach to tree competition is described in the next section.

Development of the structural models (and the modifier equations as well) essentially began with several explanatory variables and a general idea of the direction and magnitude of their effects on tree growth. The availability of derivative free non-linear estimation packages[1/] however provides the somewhat dubious capability of constructing an almost unlimited number of explicit model forms. To make the task manageable, a preliminary data sorter was delveloped to aid in model construction. As input, this program accepts a series of ranges for each potential independent variable. For each possible combination of range classes, the program performs an intersection on the data and computes the average value of the appropriate growth variable. The net effect of this procedure is to essentially hold all other variables constant and provide some indication of the effects of one variable on growth. The program was also used to examine interactions among independent variables. This screening process provided an initial basis for developing explicit model forms.

E. Modifier Development

Equation modifiers are used to incorporate two different types of "random" factors into the model system. The first is considered the "calibration" factor. It is quite unlikely that the model system

1. All non-linear parameter estimation in this report was accomplished with the IMSL subroutine ZXSSQ. This subroutine was imbedded in a larger overlay routine prepared by the authors which was used to summarize the estimation results and develop statistics comparable to the output of standard linear regression packages. A similar overlay routine utilizing the IMSL subroutine RLSTEP and related software was developed for linear least squares parameter estimation.

presented here will exactly portray the growth of any tree or group of trees. Hence, when evidence is available to suggest that the system predicts low or high, this information can be used to adjust the system and produce more precise predictions. The methodology for accomplishing this is described later.

A second type of "random" factor is incorporated to model unexplained variation in tree response. It would be somewhat heroic to expect a simple system of equations to be capable of totally explaining the development of all trees in a complex bilogical system such as a forest stand over periods spanning several decades. There will be some variation that cannot be accounted for. For example, the structural components of the increment equation for change in tree DBH squared accounts for approximately half of the total variation in the data. Hence, the unexplained portion is a substantial factor. We have found that for proper functioning of the system of models, the unexplained components of variation need to be explicitly recognized and incorporated. The reasoning for this may be clarified with the following example.

Consider a plot made up of saplings where all the trees are identical in terms of the characteristics incorporated in the models. Applying the increment equations to these trees would result in identical predictions. After a fifty year projection, all the trees would have the same predicted characteristics. However, we know this doesn't happen in practice. Trees differentiate into different crown classes presumably because some trees grow slower or faster than others due to items not specifically incorporated in the models (within-plot microsite differences, genetic variability, etc.). Competition subsequently acts to accelerate this differentiation.

The fundamental problem is that plot volumes are noticeably different if we apply a tree volume equation to the "mean tree" versus summing the volume estimates made on individual trees and then taking an average.

To circumvent this problem in forecasting, we adopt the following procedure, a variant of which was suggested by Stage (1974).

a) As part of the initialization phase in projecting plot growth with this model system, each tree record in the plot inventory is tripled and the tree weights are reapportioned so the current standing plot inventory is virtually unchanged.

b) Each of these records is then assigned percentage deviations so some of the trees are growing slower and others are growing faster than the predictions. These percent deviations are the equation modifiers for each tree and are 'permanent' for all subsequent growth projections.

By this procedure, we attempt to mimic the variation that is actually inherent in forest growth processes and obtain more realistic projections. Procedures and models for modifier assingment are given in section X.

F. Error Components

In fitting these increment equations to data, some explicit assumptions about the form of the error components in the model are necessary to develop appropriate estimation techniques, judge the adequacy of any model fit, and to suggest appropriate forms for equation modifiers.

For either total height or DBH increment (which we may in general refer to as I_{ijkl}), fitting the structural portions of the models to data produces residuals with variances being approximately proportional to the square of predictions.

This would suggest models of the following form would be appropriate.

$$I_{ijkl} = f\{x, \theta k\}\{1 + u_{ijkl}\}$$

with the u_{ijkl} term representing a proportional error rather than the "usual" additive assumption.

For analytical purposes in parameter estimation, weighting both sides of these equations with weights being approximately the inverse of of predictions can produce a transformed model with additive error terms.

$$I^*_{ijkl} = f^*\{x, \theta k\} + u_{ijkl}$$

In judging the adequacy of any fitted model, there is a tendency to associate high "R^2" statistics and low overall root mean square residuals with supposedly "good" models. We have taken the view however that the random error term can be decomposed into several factors, some of which are important to the analysis and some of which can be considered noise. Table 1 provides a description of possible main components into which the error term in these models might be decomposed. The point we wish to make is that it is desirable for these models to account for as much variation as possible in describing growth differences between plots and between trees within plots. Variation due to random periodic effects or measurement error might comprise substantial proportions of the residual variation. However, there is little that can be done to explain these sources of variation. Even if we could, this would contribute little to the explanatory power of the structural component models.

The following form was used as a model for the error term:

$$u_{ijkl} = a_{ik} + b_{ijk} + r_{ijkl} + g_{ijkl}$$

with the definitions for these terms given in Table 1. Further details and rationale can be found in Appendix II.

In estimating the structural parameter vectors in the increment models, unbiased estimates usually require some minimal assumptions

Table 1. Possible Sources of Random Variation In Tree Increment Equations - main effects

Source (symbol)	Description
Plots(a_{ik})	Mean growth on individual plots over extended time periods may depart from the regional norm. This may be due to the inadequacy of site index as a measure of productive capacity, site misclassification, or overall inadequacies of the model system.
Trees(b_{ijk})	Relative to a_{ik}, individual trees may consistently grow slower or faster than other trees. This may be due to genetic differences, soil related microsite differences, or within plot density differences not completely explained by model competiton indices. This later component may be altered by harvest operations which in itself can be considered a random component. Data limitations however have precluded specific consideration of this factor.
Periods(p_{lk})	Specific calendar periods may be associated with climatic differences significant enough to influence tree growth. Other periodic factors may be biological such as widespread occurences of seed production years when photosynthates are diverted to cone production at the expense of stem development.
Replications(r_{ijkl})	If periods are not significant soures of variation, then repeated observations on individual trees can be considered replications and short term fluctuations in tree growth can be considered replication error.
Measurements(g_{ijkl})	Unfortunately, the growth observations as well as some of the explanatory variables used in modelling were not recorded without error. It has become evident in the analytical phase that this source of variation is a major component of residual variation and needs to be explicitly recognized.

about the expected values of each random component in the error term. Usually, these assumptions are that each term has an expected value of zero and that the errors are uncorrelated with the explanatory variables. In totally controlled experiments, randomization in sample selection and assignment of experimental units to treatments is done to insure the reasonableness of these assumptions. However, in the situation we are dealing with, the majority of the data available for modelling can be classified as cross-sectional: the observations consist of single growth measurements on several trees. From this data base, the task is to construct a model capable of estimating a time series: several growth estimates on single trees. In this modelling situation, is practically impossible to randomly select a sample that is unbiased in terms of the assumptions we would like to have concerning the errors in the models. This causes some problems in parameter estimation that are attributable to both (a) stand development characteristics and (b) sample selection.

(a) Stand Development

In developing the form of an increment model, it is difficult to hypothesise some growth relationship that is devoid of size characteristics of trees (total height, crown length). When we select sample trees, these characteristics are given (e.g., we cannot assign the tree an arbitrary height or environment and observe its growth response.) If we are willing to make the assumption that some trees are inherently "better" or "poorer" than others in terms of factors not incorporated in our model such as genetic or microsite variablity (analytically, the combined variance of the a_{ik} and b_{ijk} terms is nonzero), then this "lack of randomness" will tend to make our assumptions about the expected value of the error components invalid. Intuitively, we know that in the course of timber stand development, there is a gradual decreasd in the number of trees due to mortality. To the extent that some of the "poorer" growing trees in the stand are most likely to become suppressed and die the older the stand gets, the more likely it is that the survivors are the better trees. (Darwin said something like this). Secondly, even if a "poorer" tree were to survive, it is quite unlikely for it to reach say 200 feet tall in a time span short enough for it to be classed as a young-growth tree. Consequently, while we might postulate a model with the expected value of our error terms to be zero, it is extremely difficult to find a sample of forest trees that can satisfy this criteria. In itself, this might not create unmanageable statistical problems if no tree size characteristics were used as explanatory variables. If they are, then the explanatory variables will be correlated with the "true" error terms. It is well known, especially in econometric literature (e.g. see Maddala, 1977), that application of ordinary least squares in situations such as these produce biased estimates of model parameters. The estimated parameters will reflect two entirely different growth effects: 1) a "real" effect that is postulated by the model and 2) a random effect that is due to upward shifts in the mean of the distribution of errors as trees as a whole get larger.

As a somewhat simplistic expository example, assume the tree basal area growth (CDS5) is a linear function of crown length (CL)

$$CDS5 = b_0 + b_1 CL$$

Also assume that the crowns of all trees extend to the ground and we go out and draw a random sample from existing trees stratified by ranges in crown length. In Figure 1. trees A,B,C come from the short crown length class, trees D and E from the medium class and tree F from the long length class. The solid lines indicate growth trends in the individual trees and the hyphenated line represents the overall population trajectory. We are hypothesizing that the upward shift in the population average between the short and medium class is due to say tree C either not growing fast enough to be represented in the class or dying. The same may be said about the next group. The dashed line represents the results of an ordinary least squares (OLS) fit to this data. It essentially goes through the mean of each group and reflects the crown length effect plus the shift in the error distribution. The trajectory represented by the hyphenated line is considered to be the real effect of crown length on growth.

The impacts of this scenario are a matter of interpretation. If we wanted to predict the average growth of trees of a given crown length in a population represetative of our sample data, we could use our ordinary least squares fit to accomplish this. However, if we wanted to use this model to predict the growth of a specific tree or group of trees over an extended period of time ,then the OLS model would be eroneous. In light of what we described earlier about tripling our tree records so that some trees would grow faster and some would grow slower, use of the "real" crown effect model would accomplish approximately the same thing as the OLS model: a greater proportion to the slower growing trees would die and the relatively faster growing trees would become those with longer crowns. The major differences occur when we begin to overlay harvest prescriptions on our model system. Use of the "real" model system tends to prevent over predictions of response when we do something radical like cut down all of the dominant and codominant portions of the stand and leave only intermediate and suppressed trees.

(b) Sample Selection

The problem previously described, which we think might be general in terms of growth analysis, is compounded in our specific case. The vast majority of data available for our modelling purposes has been drawn from historical records of existing permanent plots. Most of these plots are from stands 20 years of age or older with most of the detail in measurements being taken on sawtimber sized timber. Consequently, our sample is deficient in small trees and most of them tend to be intermediate or suppressed trees in stands composed of larger timber.

Remedies to both of the problems previously described have required estimation procedures that are a departure from direct application of conventional least squares. A complete estimation scenario as well as the rational and analysis for the selection of the error models is given in Appendix II.

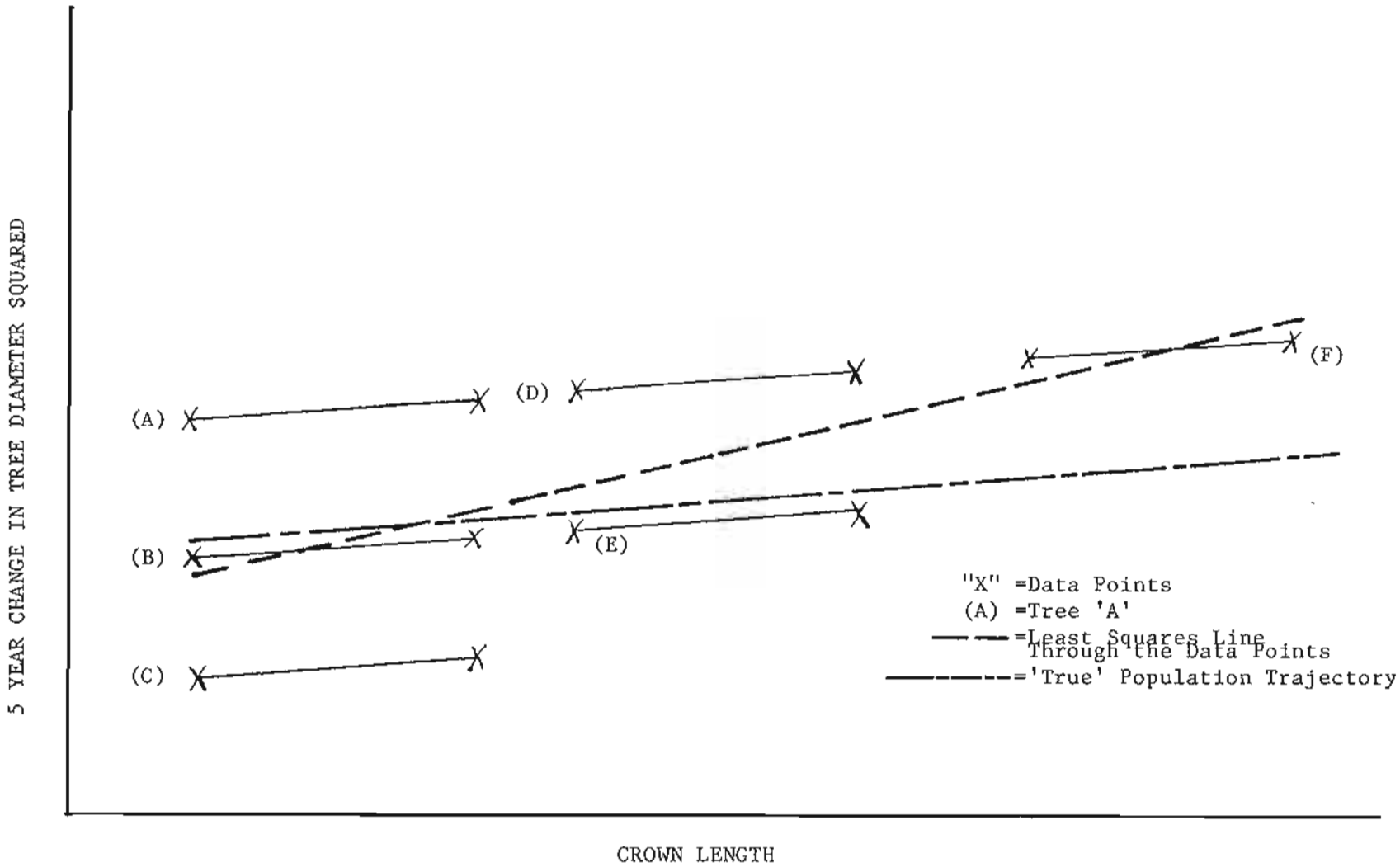

Figure 2. Illustrative comparison of a Least Squares estimate versus the presummed true Population Growth Trajectory of a Hypothetical Sample.

Earlier attempts to construct a model system ignored the problems previously described. The results were somewhat frustrating: frequent blowups would occur in attempts to simulate thinning response, the models were insensitive to items we felt should be major effects, and some of the estimated parameters had the wrong signs. The effects were often dramatic due to the exceptionally high growth rates of coastal stands. Our attempt to implement the type of model system previously described by seperating the "real" effects from the error components in parameter estimation and overlaying a "modifier" scheme where deviations assigned to each tree are permanent for the life of the simulation has produced a yield prediction system that produces reasonable results.

V. MEASURES OF COMPETITION

It is generally recognized that individual trees in dense stands grow less than their counterparts in more open stands. Similarly, in a given stand, understory trees tend to grow less than overstory trees. Both of these observations allude to the more general phenomena of inter tree competition. Historically, distance independent tree modellers have attempted to construct competition measures based on (a) a measure of stand density (basal area, stems per acre, sums of diameters, etc.) and (b) a measure of relative size (ratio of tree to average stand diameter, percentile in the diameter distribution, ratio of tree height to dominant tree height).

Our experience has indicated that it is difficlt to develope consistent and biologicaly interpretable measures of tree competition based on the density-relative size approach. Consequently, a somewhat different approach was developed for use in this model system.

Canopy cover percent is a familiar concept to foresters particularly in remote sensing applications. It is frequently expressed as the proportion of the ground area occupied by the vertical projection of tree crowns. This figure represents canopy cover at ground level. In a more general sense, if allowances are made for crown overlap, it is quite possible for the canopy cover percentage to be greater than 100%. If we begin to take "horizontal slices" through the stand at different heights, the canopy cover percent will decrease until at the tip of the tallest tree, it is zero. If canopy cover percent is expressed as a functon of height above ground, different stand structures will display different "canopy cover profiles". Figure 3 shows representative profiles for even-aged, all-aged, and two storied stands.

Intuitively then, this canopy profile provides an index of density at different heights on a given plot. It can be thought of as being related to average light availability at a given height above the ground and as such, provides some measure of competition. Before developing an exlicit competition measure, a description of the method used to quantify the canopy cover profile is in order.

A. Computation of Canopy Cover Profile

The basic information available for modelling the canopy cover

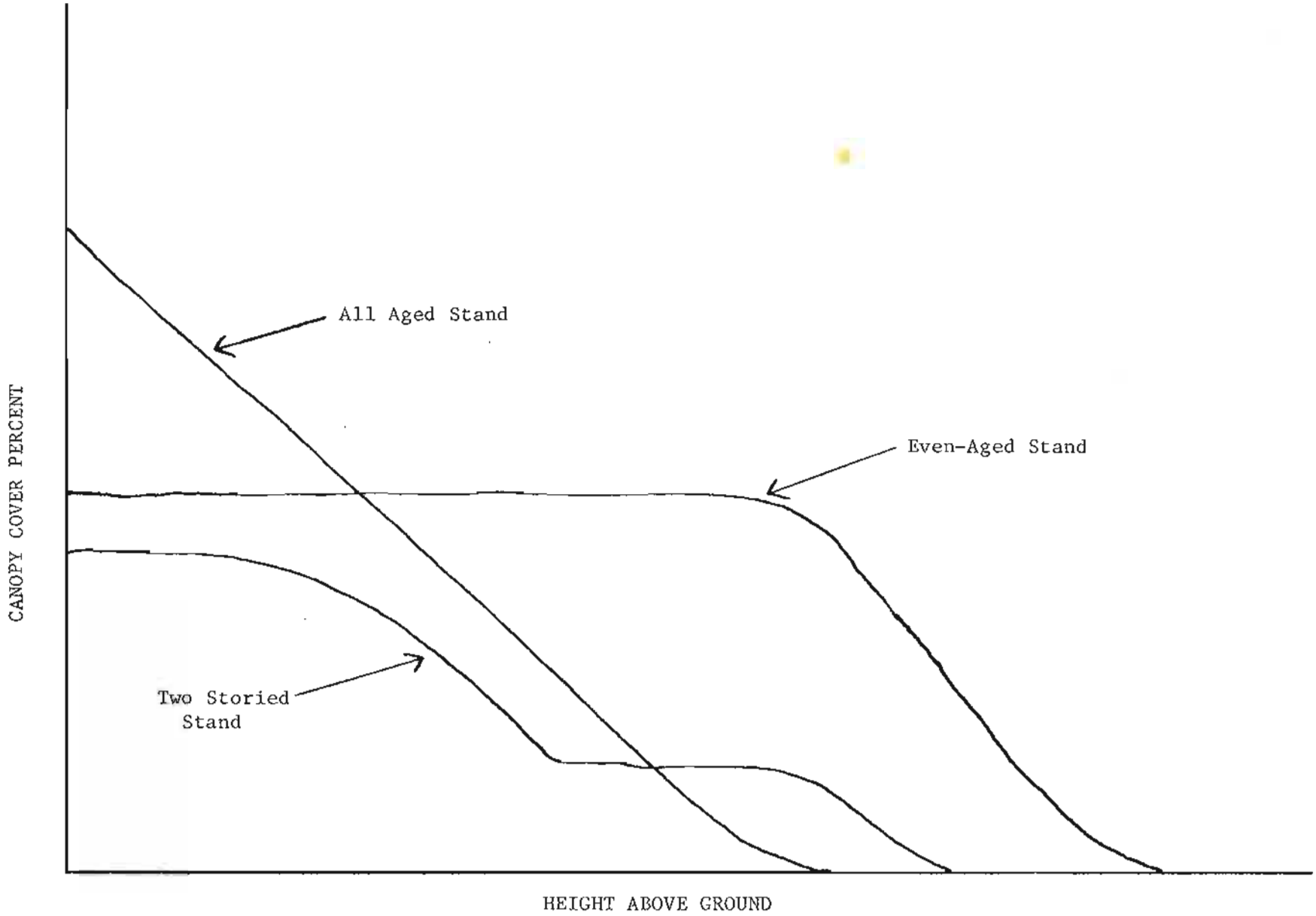

Figure 3. Illustrative Canopy Cover Profiles For Stands of Different Structure

profile consists of total height and live crown ratio of each tree on a plot. From these two variables, we may easily compute crown length and height to the crown base.

To use this information to develop a crown canopy profile, we need to know the crown radius at different points throughout the entire live crown on each tree. As this type of information is difficult and expensive to collect, we have chosen to use a model to estimate crown radii. Casual inspection of forest grown conifers indicates that the crown profile of individual trees is somewhat parabolic above the zone where crowns of adjacent trees begin to overlap. Below this point, the profile is somewhat cylindrical because branch growth is retarded due to poor light conditions and possibly mechanical effects due to branch interlocking. The lowermost branches may even be shorter than higher branches.

Mitchell (1975) developed a crown width model for Douglas fir in the Pacific Northwest. Using coefficients he provides, the following approximation can be obtained:

$$CW_i = 22.503 \ (\ln (L_i/20 + 1)) + d_i \qquad (1)$$

where

L_i = Distance in feet from tree tip to a point "i" in the tree crown

d_i = tree bole diameter in feet at point i

CW_i = crown width in feet at point "i".

This expression is only for the portion of the tree crown above the general zone of branch contact. Sufficient data were unavailable to estimate the coefficients in Equation (1) for each of the species groups we are modelling, however, a spot check with a small amount of data indicated that Equation (1) provided a fair approximation for young-growth conifers in the North Coast although there is considerable variation between trees. As the basic objective here is to develop a consistently applied index rather than an absolute measure, Equation (1) was used a basis in the following procedure for developing a crown canopy profile.

a) The "d_i" term was assumed to be zero. This introduces a slight consistent underestimate but as the canopy cover profile is used as an index, this was not considered to be a significant problem.

b) Equation (1) was applied to all eight species groups.

c) The equation was applied to the entire crown of each tree with no adjustment for possible departures below the point of branch intact.

d) The canopy cover profile takes the form of a vector with

consecutive elements representing canopy cover percent at 10 foot increments above the ground.

e) To provide estimates of this vector, equation (1) was applied to each tree to estimate crown widths at 10 foot intervals. Each of these crown widths was used to estimate crown area by assuming cross sections were round. Multiplying the areas by the tree weight divided by 43560 provides an estimate of the trees contribution to the crown canopy vector. Below the crown base, the contribution was assumed to be the same as at the crown base.

Probably the most deficient aspect of the previous procedure stems from assuming hardwood crowns are exactly like conifer crowns in contributing to tree competition. Profile dimensions and light penetration qualities are noticably different. To test wether this has a significant effect, a hardwood conopy profile was also computed for each sample plot used to develop tree increment equations and analyzed in the modelling process. Results were inconclusive due to the small number of sample plots that had significant numbers of both conifers and hardwoods. A more conclusive analysis will have to wait until better data sources become available.

B. Development of a Tree Competition Index

Our initial thought was to use the estimated canopy cover percent at a point, say, in mid-crown of each tree as a measure of competition. The crown size of a tree is directly related to its growth capabilities and the degree to which it is shaded would be a measure of how much its growth would fall short of the potential growth it could attain in an open grown or full sunlight condition.

However, using mid-crown as a reference point would presume that for two trees of a given height on an individual plot, the one with the shorter crown would be assigned a lower competition measure. In undisturbed stands, trees with relatively long crowns tend to be ones adjacent to holes in the canopy and are in a relatively lightly stocked position within the plot. Conversely, trees with shorter crowns tend to be in relatively dense positions. This apparent anomoly in the relationship between crown length and canopy density stems from not recognizing spatial arrangements of trees. As a spmpromise, we have chosen reference points independent of crown length. These points are at some proportionate amount of total tree height. While not being "perfect," it at least assigns trees of the same height within a plot the same competitive index. Explicit forms of the competitive index are detailed in the following sections.

VI. CROWN BASE MODELS

There are some situations where projections are desired for plot inventories that are deficient in the tree data required to run this model. This section describes general models that can be used to

predict missing crown measurements.

A. Uncut Natural Stands

In uncut natural stands, we have found a high correlation between crown length and density which can be exploited to give reasonable estimates of crown length. Once stands have been subjected to harvesting, this relationship becomes somewhat ambiguous and a different strategy must be used.

Model C-1

In this model, the crown canopy vector is initially estimated by assuming the crowns on all trees extends to the ground. By linear interpolation, the height at which the canopy closure is 80% is estimated. This point was chosen because the 80% canopy closure height was about the same wether we used actual crown lengths on trees or assumed the crowns on all trees extended to the ground. This value was then used in the following model:

$$HTCB = CP_{80}\{1-\exp(\alpha_0 + \alpha_1 Ht + \alpha_2 Ht/Hm)\}^{\alpha_3}$$

where

HTCB height to the crown base of the subject tree in feet

CP_{80} Height to an estimated canopy closure of 80% in feet.

Ht Total height of the subject tree in feet

Hm Average total height in feet of the largest 20% of the trees by DBH on the plot.

exp(x) 2.71828... raised to a power of 'x'

α_i Species specific coefficients estimated by non-linear regression methods

Sample trees were only selected from plots where the trees used to compute Hm were essentially dominant and codominant trees. Two storied stands were not used. Coefficient estimates and a statistical summary are shown in Table 2. Coefficients were not obtained for alder or old growth because data were unavailable. A limited amount of data was available for the the "other young growth conifer" group but the results were very similar to Douglas fir. Consequently, we suggest all young growth growth conifers other than redwood be treated as Douglas fir when using these models.

Table 2. Coefficients and statistical summary for crown base model C-1.

	α_0	α_1	α_2	α_3	$s_{y.x}$	R^2	sample size
Redwood	-.065	-.0012	-2.66	2.31	14.5	.68	1009
Douglas Fir	-.152	-.0196	-1.29	4.07	14.1	.84	488
Tanoak	-.157	-.0046	-2.79	3.72	11.0	.39	108

B. General Crown Base Models

The previous model is for even-aged natural stands. In situations where the stand structure cannot be reasonably classified as even-aged or in stands where harvesting has occured, the following model has been found to produce satisfactory results for trees in poletimber size classes and larger.

Model C-2

In this model, height to the crown base is estimated as a function of total height and tree diameter (DBH)

$$HTCB = Ht\{1 - \exp(\alpha_0 + \alpha_1 DBH + \alpha_2 Ht)\}$$

Coefficient estimates for this model are given in Table 3.

Table 3. Coefficients and statistical summary for crown base model C-2.

	α_0	α_1	α_2	$s_{y.x}$	R^2	sample size
Redwood	-.856	.0217	-.0045	11.9	.77	1409
Douglas Fir	-.639	.0275	-.0066	14.3	.81	788
Tanoak	-.957	.021	-.0049	7.6	.70	278

VII. DBH INCREMENT MODELS

A. Structural Components

The DBH increment model uses five year change in tree DBH squared (CDS5) as the dependent variable and the structural portion has the form

CDS5 = (potential)(competition factor)

1. Potential

After some exhaustive testing, relying extensively on preliminary analysis made with the data sorting program as well as actual performance tests, the "potential" portion of this model was specified to have the form

$$(\alpha_0 + \alpha_1 S)\{1 - \exp((\alpha_2 HT + \alpha_3(CL + HTG5))\}^{\alpha_4}$$

where

S = site index of the appropriate species

HT = total height in feet

CL = crown length in feet

α_i coefficients to be estimated

HTG5 = estimated future five year height growth in feet.

2. Competition Component

After testing innumerable possible functional forms, a logistic related function of canopy closure at different percentages of tree height was chosen to represent the effects of competition. This portion has the form

$$1/\{1 + \exp(\alpha_5 + \alpha_6(CC_{66} + \alpha_7(CC_{40} - CC_{66}))^{\alpha_8})\}$$

where

CC_{66} canopy closure (expresssed as a decimal) at a point equal to 66% of total tree height

CC_{40} canopy closure at 40% of tree height.

α_i coefficients to be estimated.

The logistic function was chosen because it has the property of yielding almost constant predictions over a wide range of low canopy closure levels yet still remains flexible enough to provide a reasonable competition response curve throughout the range of canopy closure values that affect tree growth.

Estimation Summary

Parameter estimates and some approximate statistics are given in Table 4.

DBH increment model coefficients were not obtained by direct application of least squares so the measures of fit are approximations. A data set not used in the estimation process, which consisted of a balanced design of multiple growth measurements on trees within plots, was used to estimate the variance components of the error term. A complete description of the data set and estimation procedures are detailed in Appendix II. These estimates are shown in Table 5. The most notable item in this table is that the combined variance estimate of the "replication-measurement" component comprises over half of the total error variance. There was no direct way to segregate this estimate into a replication variance and a measurement variance but indirect methods (see Appendix II) would suggest that the measurement variance is about 75% of the combined estimate.

B. Calibration Factor Development.

In light of the estimation problems outlined earlier estimation procedures were utilized that attempted to develop a regression surface that was most proportionatal (parellel in the transformed model) to the growth trajectories of individual trees over multiple growth periods. In design then, the model response surface represents the "average" trajectory of trees in our sample. Because of presumed shifts in the

mean of the distribution of errors, it is likely to overpredict for small trees and underpredict for larger ones as a whole. The purpose of the calibration factor is to scale the model to approximate the mean response of all trees with characteristcs similar to the appropriate tree in the plot list. In essence then, if we begin to model the growth of a tree that has managed to survive and become "large", then the fact that tree is already large indicates that the tree has superior growth attributes and probably is growing faster than our model would indicate. Conversely, the "average" tree in a young stand has less evidence to suggest it might be a fast growing tree.

The most obvious tree characteristic to use as a calibration variable is the tree's past growth rate. Indeed, this course will be pursued and reported upon later. However, in situations where there is no growth data available, current size characteristics offer one avenue for the development of calibration factors.

Tree DBH was not used at all in the development of the structural DBH increment model even though linear correlations of this variable with CDS5 were slightly less than between CDS5 and crown length. Our reason for excluding DBH from the model was twofold: 1) it is difficult to come up with a biological interpretation for this relationship and 2) in the intended use of this system (repeated solution of short term increment equations), DBH represents a special function of a "distributed lagged" form of the variable we are trying to predict. This fact would result in an additional form of bias in estimation on top of all the other problems previously mentioned.

Rather than use tree DBH as a "causal" factor in modelling, we view it as the cumulative effects of the tree growth process. If we assume that two trees of the same height and crown length on a given plot have the same competitive stress (which is the rather coarse assumption built into our model), then it would seem logical to assume that the one with the larger DBH is also growing faster in basal area. However, to extend the argument to say that the relative growth differences of two trees of the same height, and crown length growing in identical environments (the same site and competitive index as used in the model), yet situated in different stands can be indexed by differences in DBH is somwhat ambiguous. The size differences may be due to historical differences in stand treatments or development. Practically, this ambiguity is one of degree and we have attempted to develop calibration models that operate consistently at the sake of some precision.

In terms of the error model outlined earlier

$$u_{ijkl} = a_{ik} + b_{ijk} + r_{ijkl} + g_{ijkl}$$

we would like to develop explicit estimators for the combined plot and tree effects as a basis for assigning growth modifiers to each tree in the plot tree list at the start of a growth simulation. Analytically, we seek a model of the form

$$u_{ijkl} = C_d\{zd_{ijkl}, \theta d_m\} + a^*_{ik} + b^*_{ijk} + r_{ijkl} + g_{ijkl}$$

Table 4. Parameter estimates and a statistical summary for the DBH increment model.

	Redwood	Douglas Fir
α_0	59.2	28.0
α_1	1.00	1.99
α_2	.00077	.00105
α_3	-.0129	-.0138
α_4	1.25	1.40
α_5	-4.501	-10.01
α_6	3.84	10.06
α_7	.11	.33
α_8	.422	.127
R^2	.49	.64
$s_{y.x}$ (percent)	80%	61%
$s_{y.x}$ (square inches)	24.3	28.5
sample size	1228	723

R^2 statistics are based on residuals from the fitted regression in unweighted form with no adjustements for heteroscedasticity. Sample standard deviations expressed in square inches were also computed on the same basis. Standard errors expressed as a percent were computed by expressing each residual as a percent of the predicted value.

where

Table 5. Estimated variance components of the DBH increment model for a balanced data subset.

Source(component)	REDWOOD	DOUGLAS FIR
Plots(σ_a^2)	.114	.088
Trees(σ_b^2)	.189	.093
Replications plus Measurements ($\sigma_r^2+\sigma_g^2$)	.322	.251

Redwood estimates were based on 53 plots, 8 trees within plots, and 2 measurements per tree. Douglas fir estimates were based on 24 plots, 6 trees within plots and 2 measurements per tree.

C_d =auxiliary calibration function that serves to reduce the combined plot-tree variance

zd_{ijkl} =vector of explanatory calibration variables observed at the start of a growth simulation.

θd_m =vector of modifier parameters

and the definition of a^*_{ik} and b^*_{ijk} follow from

$$a_{ik} + b_{ijk} = C_d\{zd, \theta d_m\} + a^*_{ik} + b^*_{ijk}$$

In essence, we are assuming that the calibration function does nothing in accounting for the combined "replication-measurement error" random effect. One basic problem in estimating the parameter vector θd_m is that the true error term, u_{ijkl}, is unobserved. Consequently, residuals from the DBH increment model, expressed as percentages (decimal equivalents) were used instead.

In the intended use of the entire growth model system, a tree list from an inventory plot is the minimal amount of tree information needed for subsequent growth simulations. However, additional information may be available in the form of total basal area growth by species components or more detailed data on past growth performance. Which degree of input data is "best" depends on the purpose for which

projections are being made and in many cases, is limited by what has been collected in the past.

To facilitate different types of calibration data and subsequent revisions of modifier equations, our approach consists of two independent equations: one for between plot species components (plot effects) and one for within plot species components (tree effects).

1. Between Plot Calibration Model

In this analysis, one single measurement period for all available growth plots[1] were used to estimate a_{ik} (the percent deviation) for each plot with at least ten trees of the appropriate species. The following model was subsequently developed:

$$a_{ik} = \beta_{0k}(HT_{ik} - \overline{HT}_k) + \beta_{1k}(S_{ik} - \overline{S}_k)$$

where

a_{ik} = predicted mean percent deviation in CDS5 of all trees on plot i of species k.

HT_{ik} = mean total height of all trees on plot i of species k in feet.

$\overline{HT}_k$ = mean average sample plot height of species k in feet.

S_{ik} =site index of species k on plot i.

$\overline{S}_k$ =mean site index of species k.

β_{jk} = regression coefficients to be estimated.

The estimated coefficients and overall mean heights and site indices are shown in table 6.

2. Within Plot Calibration Model

Analysis of residuals expressed as percent deviations from the respective plot means has indicated that variables such as ratio of tree diameter or total height to plot means are the most hightly correlated variables, particularly in even-aged stands. However, these varibles are somewhat ambiguous as explanatory factors in multi-storied and uneven-aged stands or stands that have been harvested. As the models developed are intended to operate in any type of stand structure and are independent of age, a model with a lesser degree of precision but more consistency in stands of variable structure was chosen as a reasonable

1. This same plot set was also used to adjust coefficients in the base DBH increment models to reflect "average" growth rates of the entire region. (see Appendix II for details).

Table 6. Estimated coefficients and mean heights and site indices for the between plot calibration model.

	β_0	β_1	mean height (feet)	mean site index (feet)
Redwood	.0023	-.0031	88.5	111.6
Douglas Fir	.0029	-.0025	101.7	137.9

compromise. This model was fitted to each species component for all available growth plots in the form:

$$t_{ijkl} = \beta_{ik_0}(HDR_{ijkl} - \overline{HDR}_{ikl}) + \beta_{ik_1}(CR_{ijkl} - \overline{CR}_{ikl})$$

where

t_{ijkl} = $\hat{u}_{ijkl} - \hat{a}ik$

HDR_{ijkl} = (total height -4.5)/DBH ratio of the subject tree

$\overline{HDR}_{ikl}$ = mean plot (total height - 4.5)/DBH ratio of species k.

CR_{ijkl} = live crown ratio of the subject tree

$\overline{CR}_{ikl}$ = mean live crown ratio on the plot of species k.

$\beta_{ij_0}, \beta_{ij_1}$ = plot specific parameters to be estimated.

Coefficients for this model were estimated for each of the plots and species groups used to estimate the between plot calibration function. For each species group, the estimated coefficients for each plot were "stacked" into a single equation system and subjected to generalized linear least squares estimation procedures using the individual plot estimated variance-covariance matrices as weights, to obtain "average" values for the coefficients.[1] These estimates are shown in Table 7.

1. This procedure, sometimes call the Zellner method, is described in Maddala, 1977.

Table 7. Mean estimated coefficients for the within plot calibration model.

	REDWOOD	DOUGLAS FIR
β_0	-.205	-.265
β_1	-.266	-1.082

The data set previously described that was used to estimate variance components based soley on the structural portion of the model was once again utilized after taking into account both the structural and calibration components of the model. The estimated variance reductions are shown in table 8. The plot and tree variance components have both decreased the most noticable reduction being in the trees within plots variance estimate. As evidence that the model formulation and analysis is reasonable, we note that the replication-measurement error variance component is virtually uneffected.

Table 8. Estimated variance reductions of the DBH increment model due to calibration models.

Source(component)	REDWOOD	DOUGLAS FIR
	percent reduction	
Plots(σ_a^2)	.07	.09
Trees(σ_b^2)	.55	.47
Replications plus Measurements ($\sigma_r^2+\sigma_g^2$)	-.01	.02

Redwood estimates were based on 53 plots, 8 trees within plots, and 2 measurements per tree. Douglas fir estimates were based on 24 plots, 6 trees within plots and 2 measurements per tree.

Calibration Discussion

One problem that we anticipate in practical use of this system is using the between plot calibration function for stands that have been harvested prior to the time when plots are measured for subsequent growth projection. Harvests tend to alter the respective plot means whenever the harvest prescription is "non-random": For example a low thinning tends to favor the bigger and "better" leave trees and a "diameter limit cut" tends to leave trees which are as a whole, "poor". The use of mean height in even-aged stands for a given site and age is one possible way to provide a more refined alignment. However, age is not used as a variable in this model system and it is somewhat ambiguous in multi-storied stands. Preliminary analysis indicates that the problem is not nearly as severe in stands that have been moderately thinned from below or thinned for spacing as it is in stands that have been "severely thinned from above". In stands that have been subjected to past harvesting, we currently recommend that some actual growth data be used to effect a local calibration. In the absence of this type of information, defining site index on the basis of the trees that are standing is a currently recommended ad hoc remedy.

VIII. TOTAL HEIGHT INCREMENT MODELS

Most historical research in height growth has centered around the development of site index curves. It is generally recognized that height growth, particularly of dominant and codominant trees is much less sensitive to changes in competition and crown size than is diameter growth. Hence, the primary determinant in estimating future height growth is based on cumulative past height growth of a group of "site trees" in a given location and is called "site index". The site index models used in this study were developed for redwood in Research Note No. 5 and conversions of site index equations of other species to this model form were described in Research Note No. 11.

A. Structural Component Development

The height growth model uses five year change in total height in feet as the dependent variable. The general form of the structural component is the same as the DBH increment model

$$HG5 = (\text{potential})(\text{competition factor})$$

In addition to the possible sample biases and estimation problems previously discussed in conjunction with DBH increment models, the lack of precision in estimating height growth has created some severe problems in judging the adequacy of any postulated model. All of the measurements of height growth have been derived from successive differences in total height measurements taken on two occasions. In general, measurement techniques involved chaining ground distances and subsequently, using a hand held clinometer to measure total height. In several instances, measurements were rounded to the nearest five feet which is probably stretching the limit of accuracy of clinometers on trees in excess of one hundred feet tall. Coupled with the fact that it

is often difficult to even see the tops of trees in coastal stands and, once trees achieve heights of over 150 feet, height growth is roughly of the same magnitude as the rounding fraction, it was not too suprising to find that about 20% of the height growth mesurements were either negative or over 2.5 times the rates indicated by site index curves. However, it was the only data available. Another interesting item which has confounded any direct attempt to derive estimates of vairiance components is that on several plots, the majority of trees measured had "negative" or extremely slow height growth or were all "growing" excessively fast.

1. Height Growth Potential

Data inadequacies have prevented the use of techniques described for DBH increment models in remedying possible sample biases. Hence, we have assumed that growth patterns depicted by site curves are at least adequate in portraying the growth trajectories of dominants. We further presume, that given an age and site index, the "average" tree grows some proportional amount of a comparable site tree. As age is not used in this model, we adopt the following conventions:

(a) Our site index curves give total height in feet (HT) of dominants as a function of site index (S) and breast high age (BHA). Implicitly,

$$HT = f_h(S, BHA)$$

(b) Manipulate the basic site index equation to express age as a function of height and site index.

$$BHA = f_a(S, HT)$$

(c) For each tree, we presume it is a dominant to get an "estimated" breast high age (EBHA) by using its current height and site index.

(d) If the tree were a dominant, its five year height growth (DHG5) could be estimated as

$$DHG5 = f_h(S, EBHA + 5) - HT$$

As a whole, trees grow somewhat less than DHG5 and, and some point, reductions in crown ratios begin to have have an impact on height growth, the "potential" portion of the height growth model has the form

$$\alpha_1 DHG5/\{1. + \exp(-2.95 + \alpha_2 CR)\}$$

where

CR = live crown ratio

α_i = coefficients estimated by non-linear regression

2. Height Growth Competition Factor

The height growth competition factor uses the same approximate form as the DBH increment model only the density term in the exponent of the logistic function is a single linear function of the canopy closure at 66% of total tree height

$$1/\{1 + \exp(\alpha_3 + \alpha_4 CC_{66})\}$$

Estimation Summary

Parameter estimates and a statistical summary of the height growth analysis are shown in Table 9.

Table 9. Parameter estimates and a statistical summary for the height growth model by species.

	REDWOOD	DOUGLAS FIR
α_1	1.09	1.19
α_2	-17.30	-19.03
α_3	-1.42	-1.71
α_4	.61	.51
R^2	.14	.11
$s_{y.x}$ (percent)	.44	.39
sample size	588	374

Low R^2 values are considered to be largely a reflection of measurement error rather than a reflection of model inadequacy. In addition to measurement problems, the estimation process was confounded by an excessively lopsided sample: approximately 70% of the available sample trees were dominants or codominants. Stratifying the sample by crown class and developing separate estimates indicated the crown ratio and density related parameters were fairly uniform (after taking into account the high degree of correlation between the parameter estimates) With all other parameters fixed, the parameter d_1 can be considered a scaling factor to represent the growth rates of all trees relative to DHG5. The problem is that with most sample trees being dominants and codominants, the estimate based on all sample trees was considered to be biased upwards. Consequently, the following procedure was resorted to:

(a) The available sample, consisting of at most six trees per species per plot, was used to estimate the coefficients in the model. The observations were weighted by DHG5 to produce an approximately homogeneous residual variance.

(b) On 37 plots for redwood and 26 for Douglas fir with at least fifteen trees of the appropriate species, all of the trees had been measured for height growth. Most of these trees lacked crown size measurements so they were estimated with the models previously described. For each plot, the provisional model described in (a) above was used to predict height growth for each tree, deviations were computed as a percent, and the average percent deviation for the entire plot was subsequently estimated.

(c) The grand average was then used to adjust the original estimate of d_1. The adjusted values are what are shown Table 9. The overall net effect of the adjustment was to reduce the original estimates by about fifteen percent.

B. Calibration Factor Development

Development of height growth calibration models is analogous to that of the DBH increment model. However, an added concern is that the random variables in the height increment equation are probably not independent of their counterparts in the DBH increment models. In order to maintain reasonable relationships in the simulated height and DBH distributions after several decades of projections, possible dependencies must be maintained.

One possible strategy to account for these correlations in the calibration model would be to develop a calibration equation for the height growth models using procedures analogous to those used for the DBH model. Simulataneous procedures could subsequently be employed whereby the residuals from both equations could be utilized in estimating parameters of some appropriate bivariate relationship. This was not attempted as the height growth data is severely contaminated with measurement errors. Including the measurement error as a

proportional effect in the DBH model was considered reasonable for reasons described in Appendix II. In the case of the height increment model however, measurement error tends to increase with tree height, and growth tends to decrease. This condition plus the inordinate amount of measurement error in height growth would prevent any reasonable attempt at covariance estimation. A second practical aspect however is that when users of this system attempt to develop their own calibration factors, height increment will seldom be adequately observed. Most auxiliary data for calibration purposes will be in the form of tree and plot basal area growth. Consequently, height growth calibration models were developed as conditional functions of the DBH model residuals.

1. Between Plot Height Growth Calibration Model

In this model, we presume that the plot effect for height growth a linear function of the plot effect for DBH growth. Specifically,

$$a_{h_{ik}} = \beta(a_{d_{ik}})$$

where $a_{h_{ik}}$ is the height growth plot effect and $a_{d_{ik}}$ is the DBH growth plot effect. Both of these variables are assumed to have means of zero. Estimated mean plot deviations are unbiased but because they are estimates of a random variable, unadjusted sampling variances are biased. Consequently, least squares estimate of the parameter β using the estimated mean plot values is biased. (The bias is due to an overestimate of σ_{ad}^2). The bias can be reduced by increasing the number of sample trees on each plot. A theoretical adjustment was considered but it was abandoned because of additional data problems. Instead, median estimators, as proposed by Wald (1940) was considered appropriate. In this method, the data is ranked by the estimated mean plot deviation from the DBH model. The data is then divided into two groups with the median being the point of separation. Subgroup means are then computed as $\hat{a}_{d1}$ and $\hat{a}_{d2}$. Analogous height growth counterparts are $\hat{a}_{h1}$ and $\hat{a}_{h2}$. β is then estimated as.

$$\beta = (\hat{a}_{h2} - \hat{a}_{h1})/(\hat{a}_{d2} - \hat{a}_{d1})$$

The coefficient β was estimated with two data sets:

(1) Predictions for height growth and CDS5 were made for each tree used to develop the unadjusted height increment model. On plots with six available sample trees, average deviations for both models were taken to be estimates of height and DBH model plot effects. These estimates were centered to the overall sample mean deviation.

(2) The same procedure was followed with the plots used in adjusting the height increment model.

Both data sets produced comparable results so they were pooled and the subsequent estimate for β was .14 for redwood and .11 for Douglas fir.

2. Within Plot Height Growth Calibration Model

In this development, residuals from both height and DBH increment models were expressed as deviations from estimated plot means. We denote these estimates as $t_{h_{ijl}}$ and $t_{d_{ijl}}$ respectively. For each species, a model of the form

$$b_{h_{ij}} = \beta(b_{d_{ij}})$$

was used. A preliminary graphical and correlation analysis as well as a general lack of theoretical guidelines indicated that anything more than a linear relationship would be stretching things a bit. Unlike the JSF data subset, a sufficient number of trees with repeated height growth analysis were unavailable for analysis. Consequently, with only single paired measurements, we can only get an estimate of the combined tree effect plus replication effect plus measurement error. Three possible methods estimating the coefficient β were subsequently considered.

(1) Two-stage estimates. In this method, the within plot calibration equation for the DBH increment model was used to predict b_{dij} which was then used as an indepedent variable . This was done using coefficients for the individual plot estimates as well as the weighted "averages" shown in table 7. The results were in general poor as they indicated almost no significant relationship between height and DBH growth tree effects. We could conclude that there wasn't any significant relationship and therefore that the height and DBH increment model tree effects were independent. This wasn't considered too tenable. As a plausible reason for the lack of correlation, we might interpret the DBH calibration model as, for a given height and crown ratio within a plot, the bigger the tree in DBH (the smaller is the height-DBH ratio), the bigger is the DBH growth tree effect. However, for a given DBH and crown ratio, the taller the trees (greater height-DBH ratio), the greater is the height growth tree effect. In any event, this approach was abandoned.

(2) Least Squares Adjustments. In this method,the estimates of $t_{h_{ijl}}$ were regressed on the estimates of $t_{d_{ijl}}$ for all plots combined for the data set used in estimating the initial height growth model. Denote this estimate as $\hat{\beta}_{ols}$. The limiting value to which this estimate tends in probability (denoted as plim $\hat{\beta}_{ols}$, see Johnston, 1963) is

$$\text{plim } \hat{\beta}_{ols} = COV(t_{h_{ijl}}, t_{d_{ijl}})/VAR(t_{d_{ijl}})$$

If we are willing to make the assumption that the covariances between the height and DBH replication and measurement effects are zero and all of the random effects are identically distributed between plots, then

$$\text{plim } \hat{\beta}_{ols} = COV(b_h, b_d)/(\sigma_{bd}^2+\sigma_{rd}^2+\sigma_{gd}^2)$$

and the theoretical value of β is

$$\beta = COV(b_h, b_d)/\sigma_{bd}^2$$

Hence, in the limit

$$\text{plim } \hat{\beta}_{ols} = \beta/(1 + VR)$$

where

$$VR = (\sigma_{rd}^2 + \sigma_{gd}^2)/\sigma_{ad}^2$$

These results suggest the following estimate of β

$$\hat{\beta} = \hat{\beta}_{ols}(1 + VR)$$

where VR is estimated from variance estimates provided in Table 5. Empirical results were as follows:

Redwood: $\hat{\beta} = (.251)(1+1.704) = .68$

Douglas fir: $\hat{\beta} = (.098)(1+2.690) = .36$

(3) Median Estimators. In this method, the plots used to adjust the height growth model were utilized. Two separate sorting varaibles were used in separate trials: tree diameter and tree height. The median of each variable was used as a point of separation and median estimators as previously described for the between plot height growth calibration model were developed. For the diameter sort, the estimates of β were .54 for redwood and .30 for Douglas fir. For the height sort, the respective estimates were .62 and .37 respectively.

The last two methods of estimation, while being based on entirely different data sets were suprisingly similar. Hence, we concluded that estimates of the coefficient β of .65 for redwood and .35 for Douglas were reasonable.

One last estimate that will be needed in the modifier construction is an estimate of the variance of the within plot tree effects (σ_{bh}^2). The method used was dictated by the data that were available.

We initially assume that the largest 20% of trees in DBH on sample plots also represent the the upper 20% of the distribution of the height growth tree effects. Secondly, we assume this distribution is normal. Plotting of within plot residuals indicated that this is a reasonable assumption for Douglas fir. For redwood the distribution is somewhat skewed and a gamma distribution might be more representative but normality was assumed for practical purposes.

For each species separately, each plot used to adjust the height

growth model was sorted on DBH and the mean value of residuals centered to plot means was estimated for the largest 20% of the trees (HG20). By integrating and manipulating the normal probability density function, the following estimate of σ_{bh} can be obtained.

$$\sigma_{bh} = .2(\mathrm{sqrt}(2\pi)(HG20)/\exp\{(-1/2)(.84^2)\}$$

Individual plot estimates were weighted by the number of trees measured on the plot and averaged to form pooled estimates. The results were .19 for Douglas fir and .27 for redwood. After adjusting for differences in reporting, the estimate for Douglas fir was very close to that estimated by Mitchell (1975) for Douglas fir in the Northwest so is was concluded that the estimates were reasonable.

IX. CROWN RECESSION MODELS

Crown length-crown ratio relationships play a major role in the DBH and total height increment equations. Consequently, crown recession models are a fundamental component to the model system, particularly when yield predictions are being made for several decades. To our knowledge, there have been no direct attempts to develop crown change models with the dependent variable being change in height to the crown base. Other modelers have used indirect methods such as (1) assumed branch mortality (Mitchell,1975); (2) estimating crown ratios from other stand variables (Holdaway et.al.,1979, Daniels et.al.,1979) and (3) developing crown length estimators and partially differentiating the equation so presumed change in crown length is a function of changes in other variables such as tree height (Stage, 1974).

These attempts were probably motivated out of necessity as an adequate data base on crown recession is almost universally a scarce commodity. Our attempts to use indirect methods in modelling this aspect were abandoned because of ambiguities and inconsistencies in application. For example, if the crown ratio on individual trees is estimated as a function of stand density, a harvest wouldn't actually result in an immediate effect on crown ratio of the residual trees but the predictions would indicate it had. A direct attempt was made to model crown recession based on data derived solely from Jackson State Forest CFI plots. A description is provided in Appendix I.

Modelling crown base recession presents a challenge because of several reasonable but contradictory observations that can be made:

1. In general, crown bases are much higher in dense stands than in moderately stocked ones. Presummably then, crown recession rates were much faster in the dense stands

2. Within stands, there is presummably a gradient of light availability that decreases with height. Height to crown base in intermediate and suppressed trees is usually less than in the dominant-codominant stand fraction. So it would seem that even though the suppessed trees are under more light competiton which would tend to be positively correlated with the crown recession,

actual change in height to the crown base is less for these trees than the somewhat lesser light stressed dominants.

At the risk of oversimplification, we offer the following scenario to provide a basis for model development. Trees with long crowns tend to be more sensitive to light competition than trees with shorter crowns. The lowermost branches on long crowned trees contribute proportionately more to branch maintenance than to bole growth and do not seem to be vital to the trees existence. In shaded conditions, net photosynthesis in these branches may be negative and consequently, they are somewhat more dispensable than lowermost branches on short crown trees. Trees that are growing rapidly in height also tend to have faster rates of crown recession. Casual inspection of undisturbed evenaged stands indicates a somewhat uniform crown base line through the dominant-codominant stand portion. However, crown basis on the noticably taller trees are somewhat higher even though the overall crown length may be greater. Presummably, faster growing regions of the tree (particularly those above the main canopy) use much more water at the expense of supplying water to the lowermost branches. This may accentuate the crown base recession.

While not being a total biological representation, the following model has been found to be adequate in practice.

$$CBG5 = [\alpha_1 CL + \alpha_2 HTG5\}]/\{1 + \exp(\alpha_3 + \alpha_4 CC_{htcb})\}$$

where

CBG5 = five year change in height to crown base

CL = current crown length

HTG5 = estimated five year height growth

CC_{htcb} = estimated canopy closure percent at the crown base.

α_i = coefficients estimated by non-linear regression.

An estimation synopsis of given in Table 10.

Table 10. Estimated coefficients and summary statistics for the crown recession model.

	REDWOOD	DOUGLAS FIR
α_1	.119	.138
α_2	.128	.279
α_3	-17.1	-6.59
α_4	14.4	6.70
R^2	.31	.51
$s_{y.x}$ (feet)	2.7	2.9
sample size	357	108

X. SIMUATION INITIALIZATION AND PSEUDO-STOCHASTIC STRATEGIES

The previous description of the model system and parameter estimates (coefficients and variances) provides the quantitative material necessary for implementation of the tree growth system. This section describes the current state of recommended operating procedures.

The modifier functions used in this system are intended to represent unobserved factors which we consider to be random in real stands of trees. Without these functions (i.e., predictions of tree growth are made with the structural models only) differentiation into size classes is retarded and, most importantly, harvest responses tend to be sluggish. Secondly, the form of the modifier functions provides an analytical basis for incorporating actual past performance data in calibration to a specific stand of trees.

At this stage in the development of this model system, we feel there are a few important criteria that should be considered in the development of the equation modifiers.

Replicability. If a given tree list entered is into this model system and subsequent growth simulations are made, the results should be the same if the same process is repeated in a different computer run. Hence, while we attempt to incorporate random factors in stand development, the assignment of random factors should have the element of replicability. For lack of a better term, we refer to this as a "pseudo-stochastic" feature.

Operational Efficiency. As noted earlier in this report, a tripling process is incorporated to approximate the distribution of the unobserved tree effects in growth simulations. Essentially, we are attempting to mimic the joint distribution of two correlated random effects: the within plot random components of height and DBH growth. To adequately saturate the bivariate probability space would require much more than a tripling of tree records. However, operational computer time is directly related to the number of tree records and a lag in response time is somewhat annoying when the system is operated in an interactive mode. A more practical problem is that if the system is used to update an entire inventory, the resulting tree lists tend to grow exponentially. Since storage and manipulation of large data sets represent a limitation on models of this type, improved methods of data condensation should be investigated. Hence, our current procedure of trebling the initial tree list is a compromise that is still being investigated although preliminary tests indicate some degree of adequacy.

A. Initialization Strategies

With no actual information on plot or stand performance available, the following proceures are used as an initialization phase in simulation.

(1) All trees in the initial tree list are assigned height and DBH growth modifier value of "1". Mean height by species components are determined, the between plot DBH and height calibration factors are computed, and the resulting estimates are added to the current modifier value.

(2) The within plot calibration models are then used to make predictions of the DBH tree effects and subsequently used to estimate the height growth tree effects. These estimates are then added to the current modifier value of each tree.

(3) The within plot tree height growth effect estimates are then used to estimate the amount of variation acounted for by the height growth calibration model. This estimate is then subtracted from the estimate of the within plot height growth variance given in the last paragraph of Section 8 to obtain an estimate of the unaccounted for variation in the within plot height growth tree effects (σ_{bh}^2)

(4) The tree list is then tripled and the tree weight of each triplet is reduced by 20%, 60%, and 20% respectively. Nothing further is done to the tree DBH modifiers. The height growth modifier for the trees receiving 60% of the original weight are also unaltered. Relative to the current height growth modifier value, one tree receiving 20% of the original weight is assigned a "pseudo-stochastic" component of

$$\sigma_{bh}\{\exp((-1/2)(.84^2)|/(.2)(\mathrm{sqrt}(\pi))$$

which is the mean of the upper 20% of a normally distributed random variable with mean of 0 and variance σ_{bh}^2. This value is added to the current modifier of the tree. For the other triplet member receiving 20% of the weight, the value is subtracted.

While this procedure is somewhat simplistic, subsequent simulation tests have indicated that the resulting stand differentiating characteristics and harvest responses seem reasonable. Future analysis is being designed to provide a more objective evaluation of the procedure.

XI. PRELIMINARY EVALUATION AND FUTURE PLANS

Currently the entire model system for redwood and Douglas fir has been coded into an interactive computer program designated CRYPTOS.(see Research Note 16). Extensive tests have indicated that the model system performs quite reasonably for evenaged stands. A sample of possible results that can be generated by the model system are documented in Research Note 18. Since Research Note 18 was published, parts of this model system were modified in light of a preliminary evaluation.

Preliminary validation studies have also begun where simulated versus actual plot development aver periods of 20-35 years were compared. At this stage, the rigor of these comparisons has been limited to visual inspection. However, we feel that these initial tests have indicated a high degree of model predictability particularly in mimicing the interactions of redwood and Douglas fir in mixture.

In another experiment, the Douglas fir component of this system was tested against results that were obtained in Washington (Chambers, 1980, Wiley and Murray, 1974). The sytem produced consistent overestimates when compared to the growth tables of Chambers by about 10% in basal area and cubic foot volume. While some degree of difference is expected soley on the basis of different methodologies, Schumacher (1930) also noted that Douglas fir in California tended to be larger in DBH than in Washington or Oregon. Temporarily reducing the structural equation for CDS5 by an amount necessary to replicate the ten year basal area growth predictions of Chambers for the mean stand of his data set[1] and subsequently comparing our predictions with his over a wide range of density and site classes indicated a high degree of compliance. We feel that the results from these comparisons supported our overall model design because the data for Douglas fir came almost exclusively from mixed stands. Average plot composition of Douglas fir by basal area was about 35%.

Our future plans are to develop some objective criteria for judging the adequacy of the model system and to further test it against as much historical data as is possible. In the course of this procedure, we

1. Chambers models were for entire stands of natural Douglas fir. To convert his stand information into the necessary tree detail needed to operate the CRYPTOS model, representative tree lists were generated with the program GENR (Research Note No. 17)

will also investigate several possible methods of calibrating the model system to specific stands when some prior growth data is available and make appropriate recommendations to potential users.

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APPENDIX I

Preliminary Data Adjustments

In addition to the information described in Section II which is needed to drive the model system in applications, the plot sample data used to develop the basic growth models requires increment measurements on tree DBH, total height, and height to the crown base. With the exception of height to the crown base, this data set was acquired by differencing repeated measurements on permanent plots or (for DBH increment only) directly from increment cores after adjusting for short term changes in bark thickness.

On most of the sample plots, total height and crown size measurements were confined to a subsample. Trees that eventually became data points in model development were selected from this subsample.

Use of the canopy cover vector to develop density measures required that (at least) estimates of crown length and total height be available for each tree on the sample plot at the initial measurement. To accomplish this, several local height- DBH regression equations as well as the "generalized" height-DBH equations (see Research Notes Nos. 8 and 12) were developed for each species on each plot. For each plot, each local model was plotted against the actual data with the aid of an interactive program on a computer terminal. One equation was subsequently selected on a visual basis with the primary emphasis being on reasonableness of predictions throughout the range of diameters on the plot. For species represented by only a few trees on the subject plot, the tree samples were merged with a more abundant species. The same process was repeated with a height to crown base - total height model form.

On plots where the DBH increment measurements were made with increment cores, no attempt was made to backdate the stand to reconstruct a plausible initial measurement. Rather, past five year tree basal area growth was assumed to be equal to the next five year increment.

No plots of any kind were used that had been harvested between measurements. No plots were used that had measurement intervals less than four or greater than eight years. Plots that had been measured during the middle of the growing season were adjusted to get a "biological growth interval" on the basis of Jackson State growth study (Bawcom et. al, 1961) This adjustment was applied to DBH growth only. Most of the plots that required this adjustment were measured during the summer months. Height growth for the year was presummed to be completed by April. There were no sample plots in our data sets that had been measured during the months when annual height growth was presummed to be occuring so no adjustments were necessary. For growth intervals that were not an even multiple of five years, the interval growth measurements were linearly adjusted to give an even five year growth measurement.

A maximum of six trees of a given species on any given plot were selected as sample trees. Trees were initially sorted by species, ranked by DBH, and then given a selection priority:

1. Total height, DBH, crown length, DBH growth, and height growth were measured.

2. Same as above only height growth wasn't measured.

3. Either total height and/or crown lengths were estimated in which case the tree wasn't used as a sample tree.

An attempt was subsequently made to select three trees below the median DBH and three from above with trees of sampling priority 1 having precedent over sampling priorities 2 or 3. If this wasn't possible, an attempt was made to get either two or one tree from each side of the median. If this wasn't possible, the plot was rejected for the species being sampled.

All of the direct measurements on crown recession are coarse and limited to the Jackson State CFI plot set. Briefly, this data set is composed of approximately 140 plots that have been measured every five years since their establishment in 1958-1960. At the initial measurement, approximately half of the trees on each plot were measured for total height. At every measurement, a vigor code (based partially on crown ratio) was assigned to almost every tree on each plot. The vigor code is for a range in crown ratios of about 10 to 20 percent. On the last two remeasurements, some of these plots were subsampled for total height and had either height to the crown base or actual crown ratio's measured. Based on the last two measurements, it was found that the correlations between actual crown ratios and the midpoint crown ratio of each vigor class was quite satisfactory. Subsequently, on only those plots that had been subsampled for heights and crown ration on the last two remeasurements, all vigor codes for all five remeasurements on each tree were converted to crown ratio estimates. To each tree sampled, the actual crown ratio measurements were also added and a linear regression of crown ratio on calendar year was estimated. On trees with only two height measurements, height growth was assumed to be linear over the total twenty year time interval. For trees with three height measurements, a linear regression of height on calendar year was estimated. Using both of the estimators, crown recession trends were developed for sample trees.

APPENDIX II.

ERROR MODEL SELECTION AND ESTIMATION SCENARIO

In general, most models that are analyzed by statistical methods contain provisions for error components as a means to account for model inexactness. While not being definitive, the form in which the error components enter the model can be classified as a) additive which is the 'usual' assumption that is made; b) multiplicative in some or all of the

model coeffiicients in which case, the resulting model is referred to as a random coefficient model; or c) some combination of a and b. The form of the errror model postulated in the main report (as a multiplicative factor) can be viewed as a constrained case of a random coefficient regression model.

A. ERROR MODEL SELECTION

Initially, the decision to express the form of the error term as a multiplicative factor was based on an analysis of residuals from preliminary model forms. Subsequently, two possible decompositions of the error term were tested with primary emphasis being placed on the DBH increment model. Data limitations prevented comparable tests with the height growth model. With the definitions of the components given in Table 4-a in Section IV, these models have the the following forms:

Model A-1.

$$u_{ijl} = a_i + b_{ij} + p_l + ap_{il} + bp_{ijl} + g_{ijl}$$

The "ap" and "bp" terms represent plot-period and tree-period interactions respectively. The sets $\{a_i\}$, $\{b_{ij}\}$, $\{p_l\}$, $\{ap_{il}\}$, $\{bp_{ijl}\}$, $\{g_{ijl}\}$ are assumed to be independently distributed with zero means and variances σ_a^2, σ_b^2, etc., respectively. Variances and covariances of any two observations $\{u_{ijl}, u_{i'j'l'}\}$ follow directly from these assumptions.

Model A-2.

$$u_{ijl} = a_i + b_{ij} + r_{ijl} + g_{ijl}$$

We make the same assumptions as in A-1 only here, if periodic effects are insignificant, r_{ijl} is a replication effect and the time period 'l' can be viewed as arbitrary.

The primary purpose in analyzing these two models is twofold: 1) to estimate plot and tree variance components and 2) to determine if calendar periods are significant sources of variation. Some estimate of the plot and tree within plot variance terms are necessary to develop modifier functions. With the benefit of hindsight, we feel that most of the variation in the increment models (excluding the contribution of measurement error) can be attributed to these two sources. If model A-1 seems appropriate which would indicate that actual calendar periods are significant source of variation, then efficiencies in estimation can be made by recognizing this source. The data available for modelling was collected for growth periods from 1952 to 1979. However, each potential five year period was not equally represented with most of the data being collected in the late 60's and early 70's.

Autocorrelations

It is quite possible that the temporally sequenced random components are not independent and that some form of a serial

correlation structure might be imposed on these components. It is known that in linear models with relatively simple autocorrelation structures, ordinary least squares estimation techniques produces unbiased but inefficient parameter estimates[1]. However, variance estimates may be biased which is of concern in this analysis. Our analysis, described below, indicates that periodic effects are negligable, at least during the period for which the growth data was collected. In Model A-2, it may be that the successive "replication" (r_{ijl}) terms are not serially independent. However, testing such a hypothesis was not attempted because for even weak tests, somewhere in the order of 10 to 15 succesive five year growth measurements on each tree would be neccesary and secondly, there is no way of partitioning out the effects of the replication components from the measurement error[2].

We feel however that one serious source of autocorrelation is an artificial one and attributable to the primary collection procedures employed for most of the data: growth is estimated as the difference between two successive tree measurements. If two successive growth measurements are made on each tree (three tree measurements are made) then both growth estimates have one tree measurement in common. If the common tree measurement is "high", then the growth measurement error component will be high for the first measurement and low for the second. Some theoretical results are detailed in Section C of this appendix.

Analysis of Periodic Effects

Two separate and somewhat unrelated methods were utilized in analyzing the effects of calendar periods; one based on an analysis of increment cores and the other based on the DBH increment model applied to a data set consisting of multiple measurements on individual trees.

Increment core analysis of periodic effects

In the summer of 1977, a latitudinal transect was made across the redwood forest type in Mendocino County beginning at Fort Bragg and ending in the vicinity of North Spur. Eight sampling locations were selected; four being on ridgetops or upper slopes and four on lower slopes. Sampling locations were restricted to be in young growth forest conditions that had been undisturbed by logging. At each location, four dominant or codominant redwood and Douglas fir trees were selected and 25 year increment cores were extracted. Ring widths were measured with a dendorchronometer and tree diameters for the last 26 years were reconstructed. From each tree, five successive estimates of five year change in tree diameter squared were subsequently computed. For each species, the following model provided a basis for analysis:

1. See Maddala, G.S., Econometrics, 1977; McGraw-Hill.
2. One direct way that this could be accomplished would be to, say, have all of the sample plots measured at least two times during the same day by different personnel. If this were done, then the measurement component would be nested in the replications and a partition could be accomplished. Practically however, it is unlikely that anyone would voluntarily be willing to do this.

$$CDS5_{il} = a_{oi} + a_{1i}T + a_{2i}T^2 + P_l + e_{il} \qquad i=1,N;\ l=1,5$$

with the following side condition

$$\Sigma P_l = 0$$

where

$CDS5_{il}$ five year change in DBH squared for the ith tree during the lth time period (l=1 for 1952-1957, l=2 for 1957-1962,etc)

a_{ni} tree specific parameters to be estimated

P_l growth effect of period l (considered a fixed factor in this analysis)

e_{il} random error term

T equal to "1" for period one, "2" for period two, etc.

The principal hypothesis of interest here is whether the periodic effects are significantly different from zero

$$H_o: P_l = 0 \quad \text{for all } l.$$

With respect to the DBH increment model described in Section VII, the model here is assumed to represent both the structural component and the combined plot-tree random factors. The form is not totally equivalent to the DBH increment model. However, it does provide a plausible basis for analyizing periodic impacts and presummably because of the instruments and care made in the measurements, measurement error is negligable. This model was analyzed two ways: 1) using observations as they were recorded and 2) weighting each observation by a value inversely proportional to the average growth of the corresponding tree. This later method was done in an effort to produce approximately equal variances for each tree that were approximately proportional to predictions.

To test the hypothesis, the e_{il} were assumed to be identical and independently distributed normal random variables. The model was initially estimated as stated and subsequently without the P_k terms. "F" statistics were computed which turned out to be less than one for both methods and both species and all species combined. Hence, we concluded that periodic effects were not significant during the calendar interval that the data was collected. Expanding the model to analyze the effects of periods in different locational and topographic settings resulted in similar conclusions.

The reduced model without the periodic effects can be used to provide independent estimates of the variances of the replication components(r_{ijl}) in the DBH increment model. Estimates of the variance

from the reduced increment core model with residuals expressed as a percent of predictions (σ_e^2) is approximately equal to to the expression

$$\sigma_r^2/(1. + \sigma_a^2 + \sigma_b^2)$$

in model A-2. In additive form, the estimates for σ_e^2 were 110 for redwood and 123 for Douglas fir. In proportionate form, the estimates were .030 and .035 respectively.

DBH increment model analysis of periodic effects

An independent data set derived from the Jackson State Forest CFI plot data (JSF data) was used extensively as a means of checking the validity of assumptions and methods used in developing the DBH increment models. Data sets were developed for both redwood and Douglas fir and consisted of several plots with and equal number of trees of a given species selected on each plot. Each tree selected had two growth measurements available(the calendar periods were from the years 1960-1965 and 1974-1979). While this plot series had been measured five times, the middle measurements lacked total height measurements. Secondly, any analysis based on successive growth measurements would be confounded by measurement error serial correlations previously discussed.

In selecting candidate plots from this data set, those that had been harvested during the first or terminal growth periods were discarded. In selecting trees on these plots, candidate trees had to have all of the necessary explanatory variables used in the DBH increment model measured and had to be living at the last remeasurement. This condition was relaxed slightly for trees that had been measured for height in 1960 and once again in 1979. Heights were linearly adjusted between these two measurements to estimate heights in 1974. In selecting trees from these plots, equal numbers of qualifying trees were selected from above and below the median DBH. Sample sizes were based on the scheme giving the maximum number of tree measurements under the restriction that equal numbers of trees had to be selected from each plot. For redwood, this produced 53 plots with eight trees per plot and for Douglas fir, 23 plots with six trees per plot.

A number of approaches were used in developing the DBH increment equation. These are described in the following section. For the JSF data set (which wasn't used in the estimation procedure) the resulting model was used to make predictions of CDS5, the differences between actual and predicted values were computed as a percent of predictions and subsequently analyzed using a nested random component model. Procedures for accomplishing this have been widely discussed[1./] and basically involve the following steps:

1. Scheffe (1959) describes the motivation for these types of models and an excellent synthesis is given by Searle (1969).

1.) Partition the data in accordance with the parameter vector associated with each random component and obtain least square estimates of the reduced model.

2.) For the "residual" sums of squares associated with each reduced model derive an expression for its expected mean square in terms of the variance components.

3.) Solve the resulting system of equations to obtain variance estimates of each component.

4.) If tests of inference are to be made, Satterthwaite's[2] methods could be utilized.

Both models A-1 and A-2 were analyzed in this manner. Tests of inference were not attempted as the estimate of the variance of the periodic component (σ_p^2) in model A-1 was consistently negative or accounted for less than one percent of the total variation.

We concluded that calendar periods between 1952 and 1976 did not account for a significant amount of variation in the DBH increment model and that model A-2 would provide a reasonable basis for subsequent analyisis.

B. DBH INCREMENT MODEL - ANALYSIS AND PARAMETER ESTIMATION

As indicated in Section IV, it is hypothesized that the random components in the increment models are correlated with some of the explanatory variables and consequently, direct applications of least squares estimation techniques would result in biased parameter estimates. This concern has been a major focal point in model development because ignoring the problem results in simulated stand yield estimates that are much lower in younger age classes than all other forms of evidence would indicate and much higher in older age classes. Instead of the characteristic sigmoid or allometric shapes, the resulting yield estimates (even basal area) tended to be somewhat exponential with maximum periodic growth rates culminating at 70 to 80 years for Douglas fir and somewhere past 100 years for redwood. Hence, even without validation tests, the entire system wasn't considered believable. The rapid growth rates of coastal stands tended to accentuate the problem. Unfortunately, this hypothesis cannot be analyzed by tests based on residuals from the fitted models because by construction (at least in linear models and by analogy, in non-linear models) they are uncorrelated with the explanatory variables.

In estimation, a two-step procedure suggested by Prais and Houthakker (1955), was utilized to obtain approximate homogenous residual variances: first the structural parameter vector was estimated by ordinary least squares. The recipricols of corresponding predictions from this model were then used as weights in a subsequent estimation.

2. See F.E. Satterthwaite, "An approximate distribution of estimates of variance components" Biometrics Bull. 1946. V2, pp110-114

Usually, there was very little difference in the parameter estimates between the first and second estimation. Subsequent analysis of residuals were made in proportionate form.

In estimating the DBH increment model

$$CDS5_{ijl} = f_d\{xd, \theta_d\}\{1 + a_i + b_{ij} + r_{ijl} + g_{ijl}\}$$

we feel that parameter estimates that minimize the variance contribution of the replication effect will produce the most appropriate model. Such a model is "most" proportional to the growth trajectory of individual trees. Ignoring measurement error, if $a_i + b_{ij}$ were known for each sample tree, substituting these known values in the model and applying direct least squares estimation to minimize the remaining residual sum of squares would result in a model with these properties. However, as these terms are unknown and by hypothesis, correlated with the explanatory variables, different techniques are necesary to produce unbiased parameter estimates. Possibilities considered for accomplishing this are as follows.

1.) After transformation so that the entire error component is expressed as an additive term, taking differences in arbitrary growth observations on the same tree would effectively purge the combined tree and plot effects from the sample, as under the assumptions we have made, these factors are constant for any given tree. Unfortunately, the only data that is even marginally sufficient for this method is restricted to the JSF CFI plot data and would require discarding over 80% of the data base. Moreover, there is no apparent direct way to partition out the measurement error components which indirect evidence indicates tends to dominant this component.

2.) Another class of techniques that were considered are generally known as "instrumental variable" methods (see Maddala, 1976). In situations where the error terms are correlated with the explanatory variables, the instrumental variable technique is to find some other variable correlated with the explanatory variable yet uncorrelated with the error term and make some form of substitution. While providing a theoretical solution to the problem, we haven't been able to conceive of any variables that would reasonably satisfy the requirements.

3.) A third method is an <u>ad hoc</u> one we have developed and can be classified as an "iterative search" technique. If the least squares estimates are presumed biased and in our case, in a known direction, then we can systematically search over the surrounding parameter space and attempt to find one that is "unbiased". Procedures and criterion for this are explained in the next section.

4.) The last method considered and employed to some extent in estimation can be classed as a "data segregation" technique. If some idea of the values of the plot and tree random components are known, then the data could be stratified into "random" classes.

Effectively, we seek some variable that is highly correlated with the combined $a_i + b_{ij}$ term that can be incorporated into the model. This method (which is a special form of an instrumental variable technique) was also used as a form of evidence in analysis and will be described in the following sections.

Search Methods

In the structural portion of the DBH increment model

$$CDS5 = (A_1)(A_2)(A_3)$$

where

$$A_1 = (\alpha_0 + \alpha_1 S)$$

$$A_2 = \{1 - \exp((\alpha_2 HT + \alpha_3(CL + HTG5))\}^{\alpha_4}$$

$$A_3 = 1/\{1 + \exp(\alpha_5 + \alpha_6(CC_{66} + \alpha_7(CC_{40} - CC_{66}))^{\alpha_8})\}$$

some or maybe all of the direct least squares estimates of the parameters may be biased. Our thought was that by systematically searching through a parameter space in the vicinity of the direct least squares estimates, some exogenous comparisons at each iteration could be applied to indicate when an "unbiased" set of parameter estimates were obtained. As a criterion, we assume that the model that produces the minimum estimated replication variance is best. As most of the data contains single measurements on individual trees, there is no way to estimate this from the data used to develope the model. Instead, the JSF data sets were used and procedures previously described under the section on analysis of periodic effects were used. Both error models were used in this analysis after the inclusion of a fixed factor to represent the the mean deviation of the data set. Emphasis was placed on model A-2 as a primary check because periodic effects were considered insignificant. Even with this data set, direct estimates of the replication variance component could not be obtained. After accounting for tree and plot effects, expressions for expectations of the remaining mean square (MSERG) involved both replication and measurement variance components. However, if the measurement error is uncorrelated with everything else and unaffected by different model parameters, then differences in MSERG are a function of the replication variance estimates only. Hence, a model that produces a minimum MSERG is an indication of the "best" set of parameter estimates.

Intuitively (see figure 2 in Section IV) what we seek is a set of parameters that result in greater predictions for small trees and lesser ones for big trees. Rather than making searches independent of the data, we have employed a constrained procedure that still makes use of the sample. In this procedure, the parameters α_5-α_8 were set at their direct least squares estimates[1]. An artificial constant (δ) was

1. Initially, none of the parameters were fixed and iterative estimates were made for all of them. However, most of the differ-

substituted for the parameter α_4. In subsequent searches, different values for the constant δ were fixed, weighted two-step estimates for the parameters α_0 - α_3 were obtained, and MSERG estimated with the JSF data subset. Minimum values of MSERG occured at values of δ of 1.25 for redwood and 1.4 for Douglas fir. Ordinary least squares estimates were about 1.7 and 5.4 respectively. Also, in direct least squares estimation, α_2 was insignificantly different from zero for redwood and significant though negative for Douglas fir. Operationally, if the contribution of current estimated height growth is ignored, a zero growth prediction results when the live crown ratio (CR) has the following value[1]

$$CR = - \alpha_2/\alpha_3$$

Hence, the ordinary least squares estimates were not considered logical. Iterative estimates indicated that the "zero growth" crown ratio was about 5% for redwood and 8% for Douglas fir which seems a priori plausible.

Using resulting parameter estimates as the basic DBH increment model essentially resulted in "more believable" simulated stand yield estimates. Further, preliminary validation tests have indicated that conformance of the model to actual plot growth is quite reasonable. However, the method is admittedly ad hoc, probably inefficient, and properties of the resulting statistics are unknown. We felt there might be two different interpretations of the procedure: a) even though it was ad hoc, it seemed to work well in practice and remedied some known problems with the model; b) all of the results were an artifact of the data and we created more problems than we solved. Consequently, a different approach was attempted to see if it supported the method.

Data Segregation Methods

In this method, what is desired is some variable that is correlated

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ences in parameter estimates were accounted for in the coefficient α_7 and it resulted in estimates that were illogical. One interpretation of this parameter is that it represents part of a system of weights for density indices at different proportions of total tree height. Another interpretation is that in our sample, the taller the tree, the greater the density difference between proportionate amounts of tree height. Consequently, we felt that the differences in this parameter estimate were an attempt of the least squares algorithm to counter effect our manual attempts to "bend" the regression surface. In other words, the initial restricted parameter space wasn't restrictive enough. Deleting this term and having density effects be soley a function of the canopy closure at 66% of tree height resulted in density related parameters hardly changing at all from their direct least squares values in the subsequent iterations. Hence, the density parameters were fixed.

1. In actual operation of the model system, trees with crown ratios smaller the the "zero growth" crown ratio were presumed not to grow.

with the unobserved plot and tree effects to be used as a basis for stratifying the data. While not being perfect, tree crown class was used and an alternative model was analyzed. This model was the same as the DBH increment model only the term "1 + w(β)" was added as a multiplicative factor where w is a 1 by 3 vector having values

$[1,0,0]$ for dominants
$[0,1,0]$ for codominants
$[0,0,1]$ for intermediates
$[-1,-1,-1]$ for suppressed trees

and β is a 3 by 1 parameter vector to be estimated.

Results from refitting the model in this form were not directly comparable to previous estimates because not all trees or data sets had crown class indices taken on potential sample trees. Consequently a different and somewhat smaller sample was used to analyze this alternative model form. We will denote the previous sample as S1 and this sample as S2. To make the comparisons as simplistic as possible, the density related parameters estimated with S1 were fixed in this anlysis.

Results for redwood in this analysis were similar to those obtained with the iterative soulution. The parameter α_5 was 1.06 and the remaining parameters were sufficiently close so that predictions from crown class analysis (without the β parameters) and the iterative solution were very similar. For Douglas fir, all of the parameter estimates of β were not significantly different from zero and the remaining parameter estimates were about the same as the ordinary least squares solution. This result was subsequently attributed to the very high degree of linear correlation between crown class and height. Mostof the small trees were suppressed and all of the larger ones were dominants. As one last check, an attempt was made to redraw a sample that had equal numbers of trees in each crown class in each possible 25 foot height class for Douglas fir (The redwood sample was fairly balanced and this analysis was not done for that species). This essentially resulted in a sample with tree heights from 50-125 feet. Parameter estimates with this sample were much more comparable with the iterative solution, the β parameters were significant, and the value of α_5 was 1.54.

We interpreted these results as being supportive of the search method previously described. Discrepancies between methods and data sets were largely due to sample imbalances which were much more severe for Douglas fir than redwood.

In summary, the iterative approach was used as a basis for parameter estimation because we could utilize a much larger sample basis than the crown class segregation techniques. While the crown class parameter estimates were significant and resulted in a model with relatively more precision, this approach was not pursued for several reasons: a) the calibration models described in Section VII are intended to accomplish the same thing as adding a crown class variable to the structural portion of the model; b) in practice, we are faced with a problem of tree crown class changing over time; c) possible shifts in the mean of the the distribution of the combined tree and plot effects as trees get

older are not totally eliminated and d) estimates of crown class are somewhat subjective, inconsistent, and not totally independent of stand conditions. Checks of plots with crown class estimates before and after harvest operations indicated a substantial number of trees being reclassified into higher crown classes. We concluded that these added problems would outweigh the potential benefits of this approach.

As one last model adjustment, growth estimates were made on all trees for a single measurement of all sample plots where truncation of the DBH measurement limit was not severe enough to distort the actual within plot stocking. Missing measurements were estimated by the procedures described in Appendix I. The average percent deviation of all trees on each plot was computed, and the plot averages were then averaged. The resulting estimate was then used to adjust the coefficient estimates of α_0 and α_1. For redwood, the estimates obtained by the iterative method were reduced by 7%. For Douglas fir, the reduction was 16%. These adjusted estimates are given in Table 4 in Section 8.

C. THE MAGNITUDE OF MEASUREMENT ERROR

Possible measurement error contamination of the statistics generated in the development of increment models has been a major concern in this study because all of our indirect attempts to assess the magnitude of this factor indicate that it is a substantial source of variation. Direct assessment is impossible because it would have required all of the sample trees to be measured at least twice by different personnel.

As a means for assessing the effect of measurement error, we first consider the DBH's on a single tree at three points in time (D_1, D_2, D_3). Measurements on this tree (d_1, d_2, d_3) are assumed to be made with error w_i and can be represented as

$$d_i = D_i + w_i$$

The following minimal set of assumptions are made

1. $E[d_i] = D_i$
2. $w_i \sim IID(0, \sigma_w^2)$
3. $E[w_i^3] = 0$

The following results can subsequently be derived

$$E[d_i^2] = D_i^2 + \sigma_w^2$$

Hence, the estimate of tree diameter squared is biased. The expected value of a growth estimate of change in diameter squared using using d_1 and d_2 is

$$E[CDS5_{2-1}] = E[(d_2 + w_2)^2 - (d_1 + w_1)^2] = D_2^2 - D_1^2$$

which is unbiased. The variance of a growth estimate is

$$V[CDS5_{2-1}] = E[d_2^2 - d_1^2 - E\{d_2^2 - d_1^2\}]^2$$

$$= 8\{(D_2^2 + D_1^2)/2\}\sigma_w^2$$

and the covariance between the two successive growth estimates due to measurement error is

$$CV\{(d_3^2 - d_2^2), (d_2^2 - d_1^2)\} = -4D_2^2\sigma_w$$

or roughly one half the measurement error variance in absolute value.

As an attempt to assess the magnitude of σ_w^2, an experiment has been run twice at the University of California involving students in the undergraduate forest mensuration class. In this experiment, students were each assigned to measure DBH's on eight redwood trees from 11 to 47 inches in diameter. Instructions were designed to approximate field instructions used in permanent plot measurement (measurements were made with steel diameter tapes at a nail 4.5 feet off the ground). Sample variances were computed for each tree and were taken to be an estimate of σ_w^2. Results from the analysis of these data can be summarized as follows:

1 No significant correlations between tree size and sample variance were found at critical "alpha" levels of 0.15 or less.

2 Residuals about the sample mean of each tree tended to be leptokurtic with most of the variation consistently due to one or two aberrant measurements.

3 No evidence could be found to indicate individual students consistently under or overestimated DBH's.

4 Average sample variance over all all trees and all experiments were 0.046 although the range was from 0.006 to 0.222 for individual trees. Average sample variances for the first run of the experiment was 0.075 and the second was 0.0176.

5 These sample variance estimates and the previous tests are themselves somewhat biased and lacking in rigor due to confessed collaboration among students in attempts to get the "right" measurements even after emphasizing that there are no right measurements. However, it is currently the only available source of data that can be used to assess the magnitude of measurement error.

In an effort to indicate rough orders of magnitude, results from the increment core analysis and the JSF data were analyzed for redwood in an additive error components model. Average DBH of the increment core trees and the JSF data for redwood were approximately 20 inches. For the JSF data, mean square error of the measurement error and replication components was about 380 square inches. From the increment core analysis, an estimate of the replication variance was 110 square inches. Using a tree size of 20 inches, and a diameter measurement variance of .046, an estimate of the measurement error variance is $8(20^2)(0.046) \cong 150.$ square inches. Thus, measurement error variance is about 50% larger than the replication variance although these results must be viewed in terms of the lack of adjustments for heteroscedasticity and the independent methods employed. In the redwood sample used to fit the DBH increment model, the mean prediction was about 40 square inches. As a proportion of this amount squared, the measurement variance is $150/(1600) \cong 0.09$ From the increment analysis, the proportionate replication variance is about 0.03.

While only being indicative, these results suggest that measurement error is significant source of variation in data collected as differences in tree measurements. We suspect that the relative differences in variation between replication effects and measurements in height growth is substantially greater than that indicated for DBH increment. Consequently, in future refinements of modelling efforts of the type described in this report, we strongly suggest that added care and alternative methods of collecting data (i.e. stem analysis) be given considerable attention.