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CROWN VOLUME AND DIMENSIONAL MODELS FOR MIXED CONIFERS OF THE SIERRA NEVADA

by

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ABSTRACT

A system of equations is presented for the prediction of crown volume and width at any point along the crown. Crown volume is defined to be the space occupied by the crown. Values for the coefficients are given for each of six species: ponderosa pine (P. ponderosa), sugar pine (P. lambertiana), Douglas fir (Pseudotsuga menziesii (Mirb) Franco), incense cedar (Libocedrus decurrens Torr.), white fir (Abies concolor), and red fir (Abies magnifica A. Murr.). These equations require dbh, total height, and height to the crown base as input. If crown width is also known, an alternative method for prediction of crown volume is demonstrated.

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INTRODUCTION

Crown width has been the basis of several competition indices involving area of crown overlap of the subject tree with adjacent trees. Gerrard (1969), Bella (1969), and Arney (1973) are among the earlier users of these techniques. Krumland (1982) had additional success with a competition measure using the predicted cumulative crown cross-sectional area for the stand at two thirds of the subject trees' height. This variable provided an improved measure of competition.

Crown volume could be used in a similar way to provide a measure of a trees ability to compete. The forest canopy can be more fully characterized by a volumetric rather than a two dimensional variable. These considerations have given the impetus to this study, although this is not a competition study, it is a study of crown dimensions.

Mitchell (1975) used predicted foliage volume as a measure of competition. He had measurements on branch growth, and viewed the crown as yearly concentric layers. Other than Mitchell's (1975) work, crown volume has been rarely considered for a competition measure, possibly from lack of data. However, there have been biomass studies which predict the volume of wood in a tree crown (e.g. Phillips and Cost, 1979). This should not be confused with the current study, where crown volume is defined as the actual space taken up by the crown.

DATA

Data for this project came from the Northern California Forest Yield Cooperative's stem analysis data base. Figure 1 shows the location of the 39 clusters from which these data were collected. There were 3 plots in 31 of the clusters and 2 plots in 8 of the clusters. The clusters were taken in four cover types defined as follows:

1. Ponderosa Pine (PP)- Having at least 80% basal area in PP.
2. True Fir (TF)-Having at least 80% basal area in either red fir and/or white fir.
3. Douglas Fir (DF)-Having at least 80% basal area in DF.
4. Mixed Conifer (MC)-When none of the above types apply.

The clusters are enumerated by cover type in Table 1.

On each plot, from 3 to 6 site trees were felled and up to 7 additional trees to get a representative sample of the diameter class distribution on the plot. These felled trees were bucked beginning at a 1.5 foot stump every 16.5 feet. The height to the base of the live crown (HCB) was recorded, and the left and right crown width from the stem center was recorded , to the nearest foot, at the top of each log within the live crown. Logs are cut regardless of merchantability until approximately 15 feet from the tree tip, to obtain as much crown profile information as possible. Other information, of interest, taken at the plot includes basal area, dbh, total height, and both the Dunning and Keene crown classes (Daniel et. al., 1979). Table 2 gives the basic statistics on the major variables of interest for the data used in this study.

METHODS

The volume of each tree crown section was computed separately from the left and right radial crown measurements with Smalian's formula. The average of the left and right volumes was taken to be the crown volume. Some of the more highly correlated variables with crown volume were found to be: dbh, total height (H), height to the crown base (HCB), stand basal area (BA), and the Dunning and Keene crown classes.

The estimate of crown volume giving the lowest mean squared error (MSE) came from the following function:

$$CV = aD^{b}H^{c}CR^{d} \tag{1}$$

where,

a,b,c,d=species specific parameters, and

CR=crown ratio

=(H-HCB)/H.

Equation 1 has the desirable property of predicting a zero volume when CR is zero. Equation 1 can be viewed as a function which predicts the maximum potential CV for a tree of given dimension, and is nonlinearly modified by CR. The logarithmic transform of equation 1 can be fit with standard least squares methods, also.

A second equation has been adapted from work by Van Deusen et. al. (1982) to allow prediction of crown volume at any point along the stem:

$$V(h) = CV - CV\left[\frac{(H-h)}{(H-HCB)}\right]^{k} \tag{2}$$

where,

h=height of interest such that HCB is less than or equal to h , and h is less than or equal to H,

V(h)=the volume at height h, and

k= a species-specific parameter.

Examination of equation 2 reveals that it is constrained to predict zero volume when h equals HCB, and to predict full crown volume when h equals H. Equation 2 can also be viewed as the following integral of the square of crown width at height y {CW(y)}:

$$V(h)= \frac{\pi}{4}\int_{HCB}^{h} CW(y)^2\, dy \qquad (2a)$$

A function to predict crown width squared at any height can therefore be obtained by differentiating (2a):

$$CW^2=4/\pi (d/dh)V(h)$$

$$=\frac{4k\ CV(H-h)^{(k-1)}(H-HCB)^{-k}}{\pi} \qquad (3)$$

Equation 3 logically predicts zero crown width at the tip (i.e when h=H). Equation 3 also implicitly assumes that the crown width increases monotonically as you move from the tip to the crown base.

Equation 3 can also be rearranged to predict crown volume above a point where crown width is known:

$$CV(h)= \frac{\pi}{4k}\, CW^2 (H-h)^{(1-k)} (H-HCB)^k \qquad (4)$$

Equation 4 should provide a more accurate estimate of crown volume if the required crown dimensions are available.

RESULTS AND APPLICATION

Equation 1 was fit to ponderosa pine (PP), sugar pine (SP), douglas fir (DF), incense cedar (IC), white fir (WF), and red fir (RF) both separately and for all species combined 2/. Analysis of covariance revealed significant differences among the species at the .05 level. This difference could not be attributed to a single parameter, therefore, Table 3 contains the parameters and summary statistics for each species. The parameter in equation 2 was also found to vary significantly by species, and is presented in Table 4.

To demonstrate the use of these functions, suppose we wish to predict the crown dimensions of a sugar pine with a 20 inch dbh, 100 foot total height, and a crown base at 40 feet. This means that the crown ratio is 0.6, since the crown length is 60 feet. Solving for CV by using the SP coefficients in Table 3, we get:

$$CV = a20^{b}100^{c}.6^{d}$$

$$= 9106 \text{ cubic feet}$$

Now this volume can be apportioned along the crown using equation 2, and the crown width at any height can be estimated using equation 3. A graphical display of this hypothetical tree crown is given in figure 2.

2/ Actually, the logarithmic transform of equation 1 was fit to more nearly meet the regression assumption of homogeneous variance.

To continue this example, suppose we know that the crown width at 40 feet is 20 feet. Therefore, this tree is wider at the crown base than figure 2 indicates. We can now use equation 4 to predict a new estimate of volume with this knowledge of crown width. The SP value of k is 1.976 giving:

$$CV = \frac{\pi}{4k} 20^2 (100-40)^{(1-k)} (100-40)^k$$

$$= 9539 \text{ cubic feet}$$

This new estimate of crown volume is about 14.5 percent greater than the original estimate when crown width is unknown. This result is logical, but it should be remembered that no constraint exists to logically relate equations 1 and 4. They are two unrelated techniques for crown volume prediction which may prove useful.

DISCUSSION

The ability to predict crown volume may prove valuable for future studies of competition. Silviculturists and those interested in crown biomass may also find these equations useful. In this study, crown volume was found to be more predictable than crown width in terms of mean squared error (MSE). Crown width has been the basis of many recent competition indices. The three dimensional nature of crown volume should more fully characterize a trees ability to compete.

It may be disturbing to some that basal area is not used as a predictor of crown volume. In fact, this analysis showed basal area to be nearly as good a predictor as crown ratio, for the data studied. This is because height to the crown base and basal area are highly correlated. Cole and Jensen (1982) found that as stand basal area increases, average tree HCB moves up the stem. However, crown ratio provides more information about an individual tree crown than stand basal area. Crown ratio is used directly to constrain equation 1 in much the same way that a competition function constrains a potential tree growth function. If stand crown volume were being predicted, then stand basal area would be more useful than crown ratio.

The function for predicting crown volume above any height may also have application with future competition indices. The volume of the crown could give an indication of tree growth potential or foliage volume. Equation 2 provides for the use of competition functions based on crown width, should this prove necessary.

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TABLE 1. Clusters by cover type

Type	Number of Clusters
Ponderosa Pine	3
Mixed Conifer	22
True Fir	10
Douglas Fir	4
total	39

TABLE 2. Statistics on the major variables of interest, overall and by species.

VARIABLE	MEAN	STANDARD DEVIATION	MINIMUM VALUE	MAXIMUM VALUE
species=all observations=986				
basal area	234.5	75.0	20.0	435.0
dbh	17.0	6.2	5.5	36.3
total height	85.2	24.9	23.8	153.1
ht to crown base	36.0	15.3	0.0	104.0
crown volume	7230.2	7191.5	51.7	58084.3
species=ponderosa pine observations=239				
basal area	243.8	66.6	20.0	370.5
dbh	19.0	6.5	5.5	36.1
total height	99.0	24.5	33.7	153.1
ht to crown base	44.0	15.0	4.5	82.0
crown volume	8151.8	7140.9	51.7	45332.7
species=sugar pine observations=85				
basal area	266.3	69.9	120.5	435.0
dbh	19.8	6.3	6.2	32.1
total height	88.8	20.6	36.7	137.7
ht to crown base	38.3	13.6	4.5	70.5
crown volume	8470.5	6498.2	227.0	27364.2
species=douglas fir observations=86				
basal area	223.6	78.9	20.0	435.0
dbh	14.6	5.8	5.6	36.3
total height	59.1	19.0	23.8	105.3
ht to crown base	22.6	10.0	4.5	41.0
crown volume	3962.7	5024.1	76.3	24414.0
species=incense cedar observations=197				
basal area	188.6	61.7	54.6	368.6
dbh	15.4	6.0	5.8	32.1
total height	86.0	24.2	38.8	150.6
ht to crown base	34.6	15.2	0.0	104.0
crown volume	10253.4	9677.9	471.2	56460.0
species=white fir observations=336				
basal area	238.8	76.0	20.0	435.0
dbh	16.3	5.7	5.6	34.3
total height	80.9	22.5	25.5	133.3
ht to crown base	34.6	14.2	2.0	72.0
crown volume	5582.7	5468.9	97.4	58084.3
species=red fir observations=43				
basal area	317.7	26.6	292.6	357.3
dbh	17.8	4.3	7.8	27.8
total height	83.9	13.5	41.5	101.3
ht to crown base	31.4	12.0	4.5	49.0
crown volume	5214.1	3771.3	295.9	14259.0

TABLE 3. Parameter values and Sy.x 1/ equation 1; CV=a*D**b*H**c*CR**d.

Species	a	b	c	d	Sy.x	n of trees
PP	10.284	1.928	.424	2.035	3491.3	239
SP	.250	1.290	1.622	1.627	3201.9	85
DF	.233	.919	1.897	1.890	1491.6	86
IC	1.081	1.083	1.465	1.168	3775.9	197
WF	6.045	1.414	.787	1.480	2105.8	336
RF	.175	2.026	1.203	2.166	3199.8	43
ALL	.926	1.021	1.511	1.716	3254.7	986

1/ Sy.x is the square root of the sum of the squared errors about the regression line divided by the degrees of freedom.

TABLE 4. Parameter value and summary statistics for equation 2; V(h)=CV-CV*((H-h)/(H-HCB))**k.

Species	k parameter	Sy.x	number of observations
PP	1.868	1090.2	426
SP	1.976	770.6	169
DF	2.063	422.0	111
IC	1.938	1365.4	317
WF	2.152	566.6	511
RF	2.042	454.3	45
ALL	1.953	954.6	1579

Figure 1. Location of stem analysis plots.

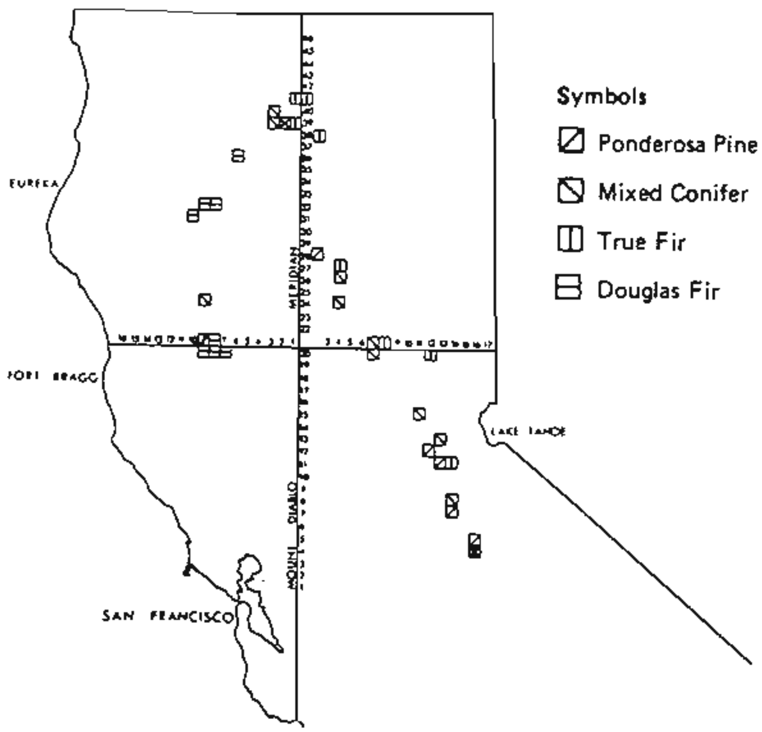

FIGURE 2. A predicted sugar pine profile for a tree with a 20 inch dbh, 100 foot total height, and crown ratio of 0.6. Total crown volume equals 9106 cubic feet.

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                                          Partial crown      Crown
                                       volumes (ft cubed) width (ft)

        100               -                  ---------       ------
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         90           ----------                8842           8.2



         80          -------------              8067          11.4



         70         ---------------             6791          13.9



         60        -----------------            5019          16.0
   F
   E
   E
   T     50       -------------------           2755          17.9



         40       -------------------              0          19.5
                         : :
                         : :
                         : :
                         : :
         30              : :
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         20              : :
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